Contents

Surds

Things to remember:

- √ means square root;
- To simplify surds, find all its factors;
- To rationalise the denominator, find an equivalent fraction where the denominator is rational.

Questions:

1. Work out

$$\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$$

Give your answer in its simplest form.

expand brackets

$25 + 5\sqrt{3} - 5\sqrt{3} - 3$

gather like terms

$25 - 3 = 22$

$\frac{22}{\sqrt{22}} \times \frac{\sqrt{22}}{\sqrt{22}} = \frac{22\sqrt{22}}{22} = \sqrt{22}$

$\sqrt{22}$ ....

(Total 3 marks)

2. (a) Rationalise the denominator of $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sqrt{3}/3$ ....

(1)

(b) Expand $(2+\sqrt{3})(1+\sqrt{3})$

Give your answer in the form $a + b\sqrt{3}$ where *a* and *b* are integers.

$2 + \sqrt{3} + 3 + 2\sqrt{3}$

$5 + 3\sqrt{3}$

$5 + 3\sqrt{3}$ ....

(2)

(Total 3 marks)

3. (a) Rationalise the denominator of $\dfrac{1}{\sqrt{7}}$

$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

$\sqrt{7}/7$ ....

(2)

(b) (i) Expand and simplify $(\sqrt{3} + \sqrt{15})^2$
Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

$(\sqrt{3} + \sqrt{15})(\sqrt{3} + \sqrt{15})$

$3 + \sqrt{45} + 15 + \sqrt{45}$

$18 + 2\sqrt{45} = 18 + 2\sqrt{9}\sqrt{5} = 18 + 6\sqrt{5}$

$18 + 6\sqrt{5}$ ....

(ii) All measurements on the triangle are in centimetres.
ABC is a right-angled triangle.
k is a positive integer.

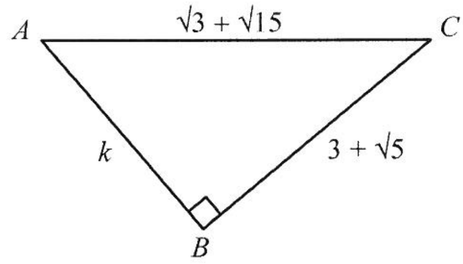


Diagram **NOT** accurately drawn

Find the value of *k*.

Pythagoras

$(3 + \sqrt{5})^2 + k^2 = (\sqrt{3} + \sqrt{15})^2$

$(3 + \sqrt{5})(3 + \sqrt{5}) + k^2 = (\sqrt{3} + \sqrt{15})(\sqrt{3} + \sqrt{15})$

$9 + 3\sqrt{5} + 5 + 3\sqrt{5} + k^2 = 3 + \sqrt{45} + 15 + \sqrt{45}$

$14 + 6\sqrt{5} + k^2 = 18 + 2\sqrt{45}$

$= 18 + 6\sqrt{5}$

k = 2 ....

(5)

(Total 7 marks)

$k^2 = 4$

$k = \pm 2 \quad k = 2$

4. Expand and simplify $(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})$

$= 3 - \sqrt{6} + 2 - \sqrt{6}$

$= 5 - 2\sqrt{6}$

$5 - 2\sqrt{6}$ ....

(Total 2 marks)

5. (a) Write down the value of $49^{1/2}$ $= \sqrt{49}$

± 7 ....

(1)

(b) Write $\sqrt{45}$ in the form $k\sqrt{5}$, where *k* is an integer.

$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

$3\sqrt{5}$ ....

(1)

(Total 2 marks)

6. Write $\dfrac{\sqrt{18} + 10}{\sqrt{2}}$ in the form $a + b\sqrt{\cancel{3}}$ 2 where *a* and *b* are integers.

$$\frac{\sqrt{18} + 10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{36} + 10\sqrt{2}}{2}$$

$$= \frac{6 + 10\sqrt{2}}{2}$$

$$= 3 + 5\sqrt{2}$$

a = 3 ....

b = 5 ....

(Total 2 marks)

7. Expand and simplify $(2+\sqrt{3})(7-\sqrt{3})$
Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

$(2+\sqrt{3})(7-\sqrt{3})$

$21+7\sqrt{3}-3-2\sqrt{3}$

$=18+5\sqrt{3}$

$18+5\sqrt{3}$ ....

(Total 3 marks)

8. Rationalise the denominator of $\dfrac{(4+\sqrt{2})(4-\sqrt{2})}{\sqrt{7}}$
Give your answer in its simplest form.

Diff 2squares!

$\dfrac{(4+\sqrt{2})(4-\sqrt{2})}{\sqrt{7}} = \dfrac{16-2}{\sqrt{7}} = \dfrac{14}{\sqrt{7}}$

$\dfrac{14}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{14\sqrt{7}}{7} = 2\sqrt{7}$

$2\sqrt{7}$ ....

(Total for question = 3 marks)

9. Show that $\dfrac{(4-\sqrt{3})(4+\sqrt{3})}{\sqrt{13}}$ simplifies to $\sqrt{13}$

again difference of 2 squares form!

$(4-\sqrt{3})(4+\sqrt{3}) = 16-4\sqrt{3}-3+4\sqrt{3} = 16-3=13$

$\dfrac{13}{\sqrt{13}} \times \dfrac{\sqrt{13}}{\sqrt{13}} = \dfrac{\cancel{13}\sqrt{13}}{\cancel{13}} = \sqrt{13}$

(Total for question = 2 marks)

Algebraic Proofs

Things to remember:

- Start by expanding the brackets, then factorise.
- Remember the following:
    1. 2n → even number
    2. 2n + 1 → odd number
    3. a(bn + c) → multiple of a
    4. Consecutive numbers are numbers that appear one after the other.

Questions:

1. (a) Expand and simplify x(x + 1)(x − 1)

diff 2 squares, again!

$x(x^2-1)$

$=x^3-x$

$=(x^2+x)(x-1)$

$=x^3+x^2-x-x^2$

$=x^3-x$

x^3-x ....

(2)

In a list of three consecutive positive integers at least one of the numbers is even and one of the numbers is a multiple of 3

n is a positive integer greater than 1

(b) Prove that n³ − n is a multiple of 6 for all possible values of n.

model 3 cons. nos. as $(n-1), n, (n+1)$

$\underbrace{(n-1)(n+1)}_{\text{diff 2 squares}}$

(n^2-1)

$(n^2-1)(n)$

$=n^3-n$

So 3 cons nos. can be modelled by n^3-n

3 cons nos. has mult. of 2 (even) and mult. of 3, so is a mult. of 6

(2)

2^{61} ~~261~~ − 1 is a prime number.

(c) Explain why ~~261 + 1~~ $2^{61}+1$ is a multiple of 3

$2^{61}-1, 2^{61}, 2^{61}+1$

a) 3 consec. numbers

(2)

(Total for question = 6 marks)

$2^{61}-1$, 2^{61}, $2^{61}+1$

prime

mult of 2

must be a mult of 3

2. Prove that

$(2n + 3)^2 - (2n - 3)^2$ is a multiple of 8

for all positive integer values of n.

expand brackets

$4n^2 + 12n + 9 - (4n^2 - 12n + 9)$

$= 4n^2 + 12n + 9 - 4n^2 + 12n - 9$

$= 24n$

24 is a multiple of 8,

so 24n is a multiple of 8

(Total for Question is 3 marks)

3. (a) Expand and simplify $(y - 2)(y - 5)$

$y^2 - 2y + 10 - 5y$

$y^2 - 7y + 10$

$y^2 - 7y + 10$ ....

(2)

*(b) Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

expand brackets

$4n^2 + 4n + 1 - (2n + 1)$

$= 4n^2 + 4n + 1 - 2n - 1$

$= 4n^2 + 2n$

$= 2(2n^2 + n)$

2 is a factor

so the number is even

(3)

(Total for Question is 5 marks)

4. * Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

model 2 consecutive integers
as x, $x+1$

$(x+1)^2 - (x)^2$

$x^2 + 2x + 1 - x^2$

$2x+1$

$x + x+1$

$= 2x+1$

(Total for Question is 4 marks)

5. (a) Factorise $x^2 + 7x$

$x(x+7)$

(1)

(b) Factorise $y^2 - 10y + 16$

$(y-8)(y-2)$

(2)

*(c) (i) Factorise $2t^2 + 5t + 2$

$(2t+1)(t+2)$

(ii) t is a positive whole number.
The expression $2t^2 + 5t + 2$ can never have a value that is a prime number.
Explain why.

in addition to the factor pair t, 1
it has the factor pair $2t+1$, $t+2$

(3)

prime numbers have only 1 factor pair

(Total for Question is 6 marks)

6. (a) Factorise $3t + 12$

$3(t+4)$

(1)

(b) (i) Expand and simplify $7(2x + 1) + 6(x + 3)$

$= 14x + 7 + 6x + 18$

$= 20x + 25$

$20x + 25$

(ii) Show that when x is a whole number

$7(2x + 1) + 6(x + 3)$

is always a multiple of 5

$= 20x + 25$

$= 5(4x + 5)$

since 5 is a factor, the number is a multiple of 5

(3)

(Total for Question is 4 marks)

7. Prove that $(n - 1)^2 + n^2 + (n + 1)^2 = 3n^2 + 2$

expand brackets

$\cancel{n}^2 - 2n + 1 + n^2 + n^2 + 2n + 1$

combine like terms

$= 3n^2 + 2$

(Total for Question is 2 marks)

8. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

see Q4

(Total for question is 4 marks)

9. The product of two consecutive positive integers is added to the larger of the two integers. Prove that the result is always a square number.

model 2 consecutive positive integers as

$x \quad , \quad x+1$

$x(x+1)$

$= x^2+x$

larger of 2 numbers

now, add to $x+1$

$x^2+x+x+1$

$= x^2+2x+1$

(Total for question = 3 marks)

Factorise

$(x+1)^2$

factor squared, hence a square number

10. Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

see Q 3b

(Total for question = 3 marks)

Transformations of graphs

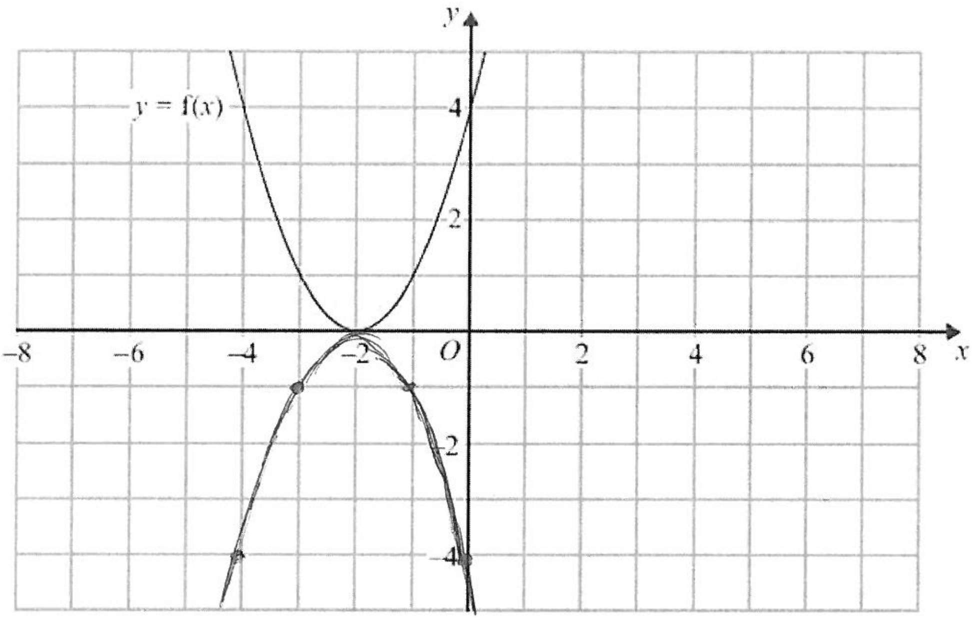
Things to remember:

1. f(x) means the function of x.
2. -f(x) is a reflection in the x-axis.
3. f(-x) is a reflection in the y-axis.
4. f(x – a) is a translation in the x-axis, a units.
5. f(x) + b is a translation in the y-axis, b units.
6. cf(x) is an enlargement in the y-axis, scale factor c.
7. f(dx) is an enlargement in the x-axis, scale factor $\frac{1}{d}$.

Questions:

1. $y = \mathrm{f}(x)$

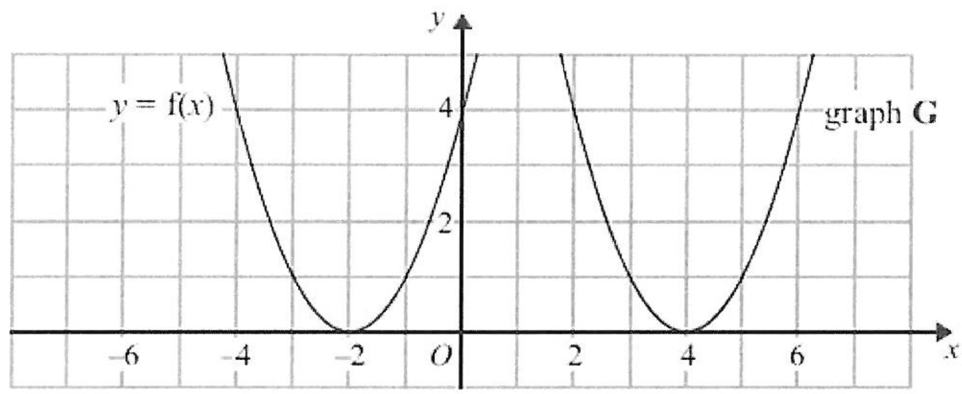
The graph of $y = \mathrm{f}(x)$ is shown on the grid.



(a) On the grid above, sketch the graph of $y = – \mathrm{f}(x)$.

(2)

The graph of $y = \mathrm{f}(x)$ is shown on the grid.



The graph **G** is a translation of the graph of $y = \mathrm{f}(x)$.

(b) Write down the equation of graph **G**.

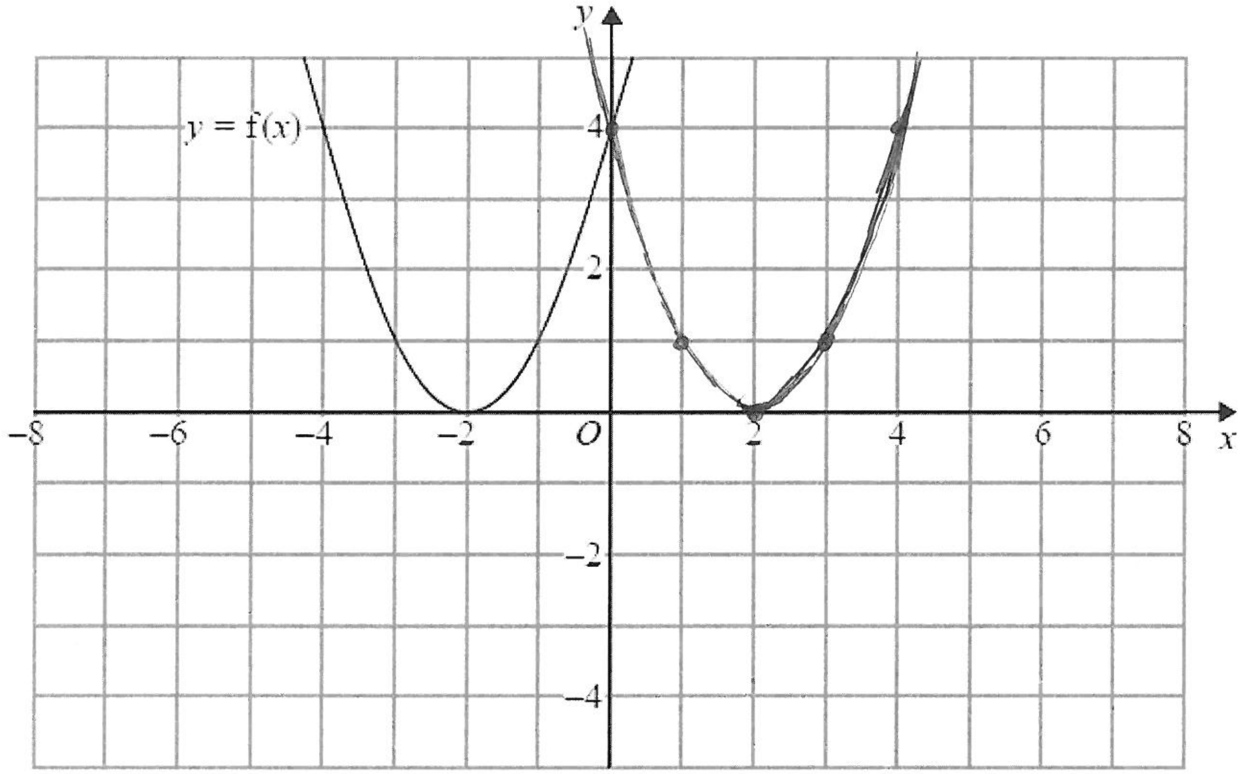
translation in the x-axis
6 units

$y = f(x-6)$

(2)

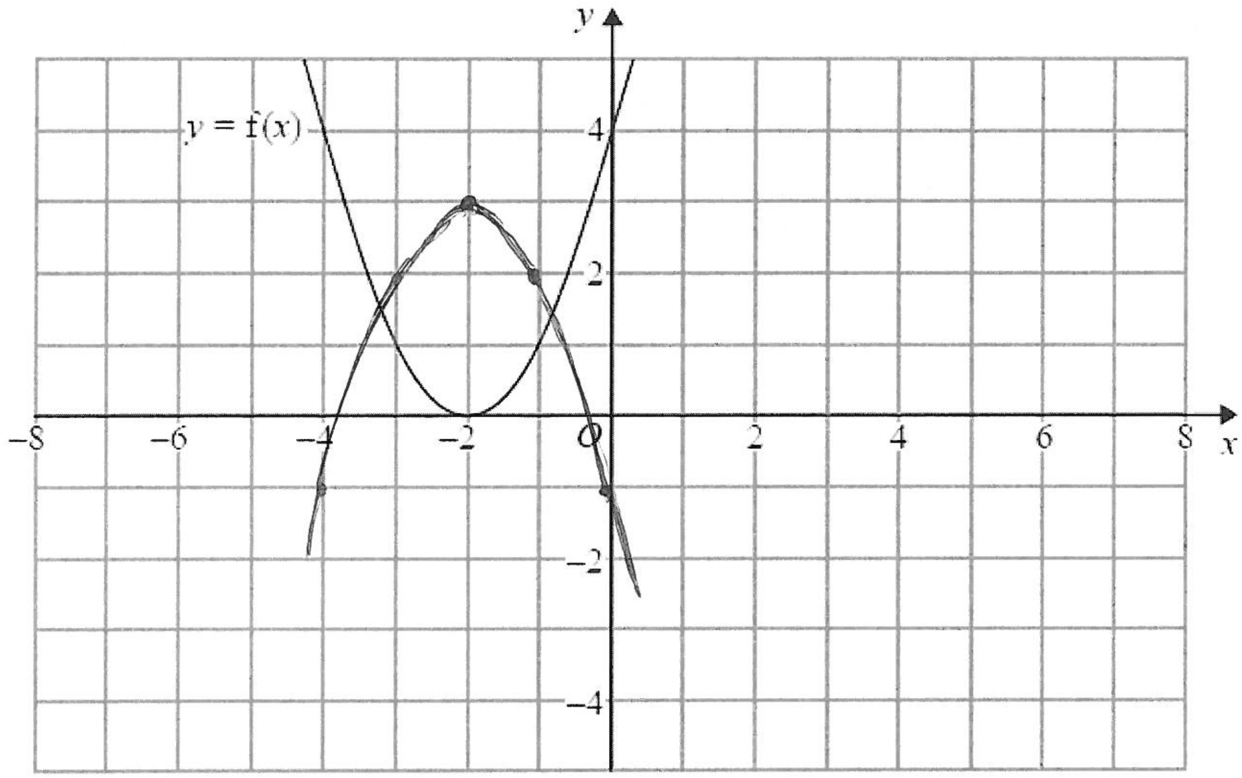
(Total for Question is 3 marks)

2. The graph of $y = f(x)$ is shown on both grids below.



(a) On the grid above, sketch the graph of $y = f(-x)$ reflection in y-axis

(1)



(b) On this grid, sketch the graph of $y = -f(x) + 3$

(1)

(Total for question = 2 marks)

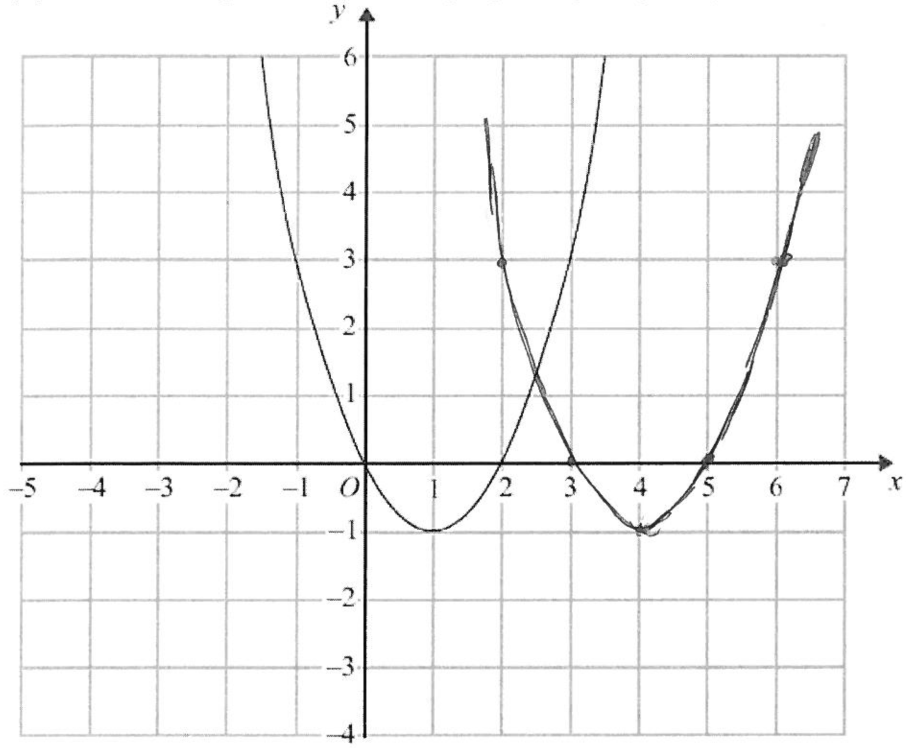
reflection in x-axis

translation $\cancel{\begin{pmatrix} 3 \\ 0 \end{pmatrix}} \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

3. The graph of $y = f(x)$ is shown on each of the grids.

(a) On this grid, sketch the graph of $y = f(x - 3)$

translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

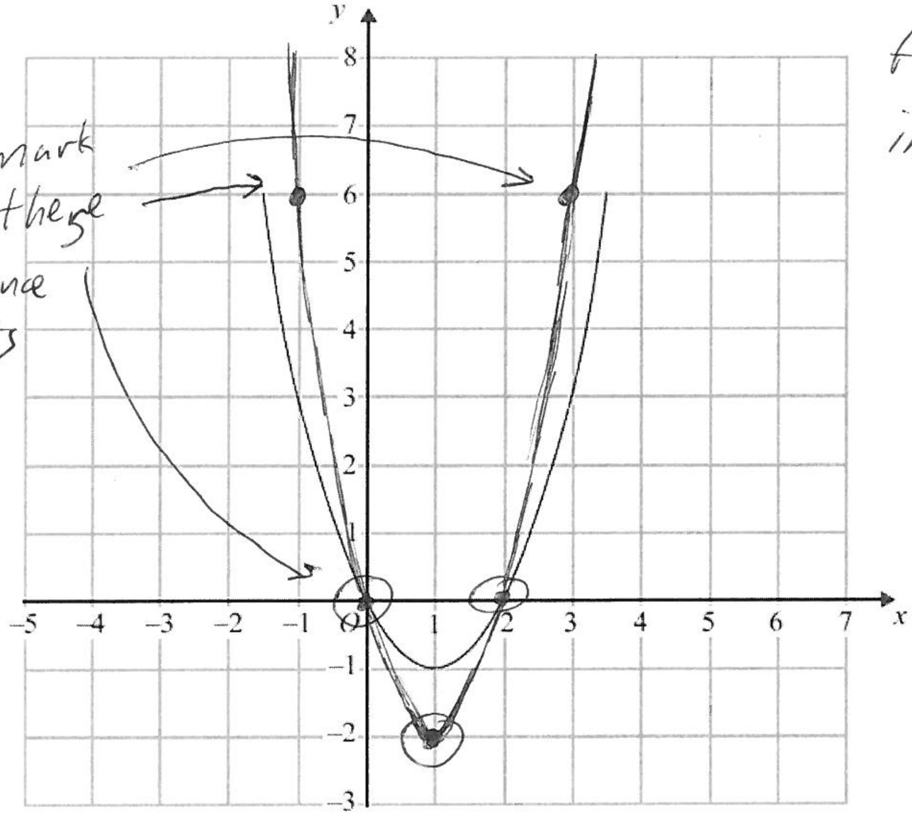


(2)

(b) On this grid, sketch the graph of $y = 2f(x)$

enlargement/stretch
factor ×2
in y-direction

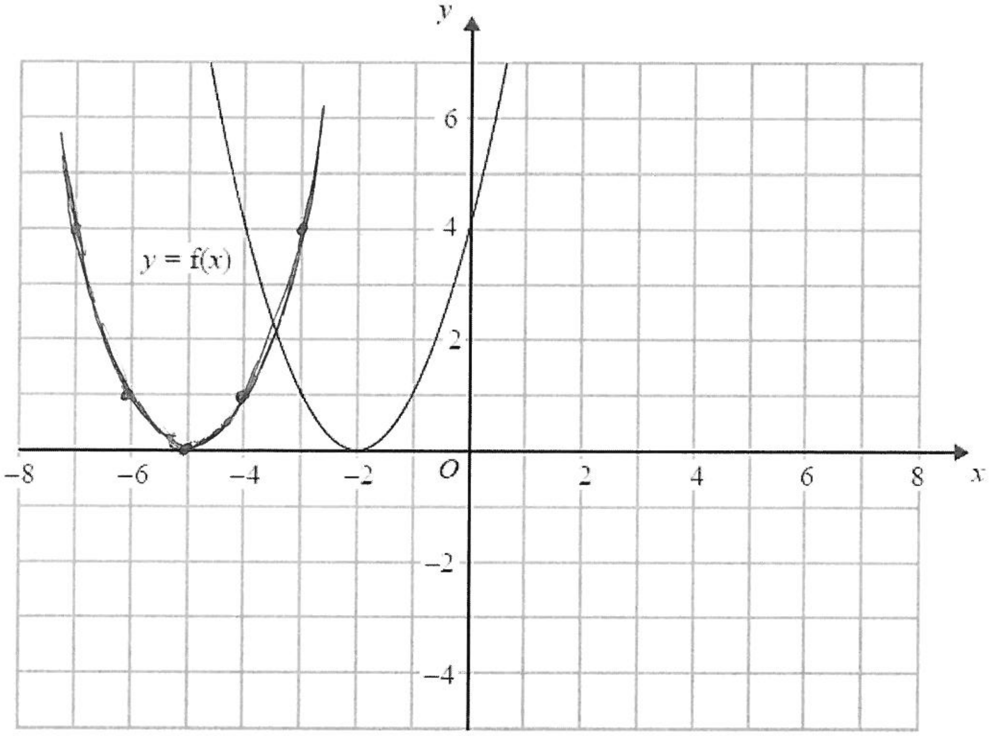
mark these
reference
points



(2)

(Total for Question is 4 marks)

4. The graph of $y = f(x)$ is shown on the grid.

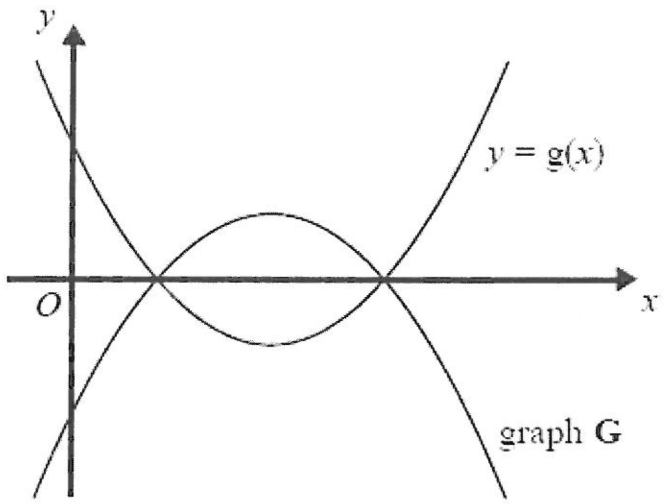


translation $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

(a) On the grid above, sketch the graph of $y = f(x + 3)$

(2)

The graph of $y = g(x)$ is shown below.



The graph **G** is the reflection of $y = g(x)$ in the x-axis.

(b) Write down an equation of graph **G**.

$y = -g(x)$

(1)

(Total for question = 3 marks)

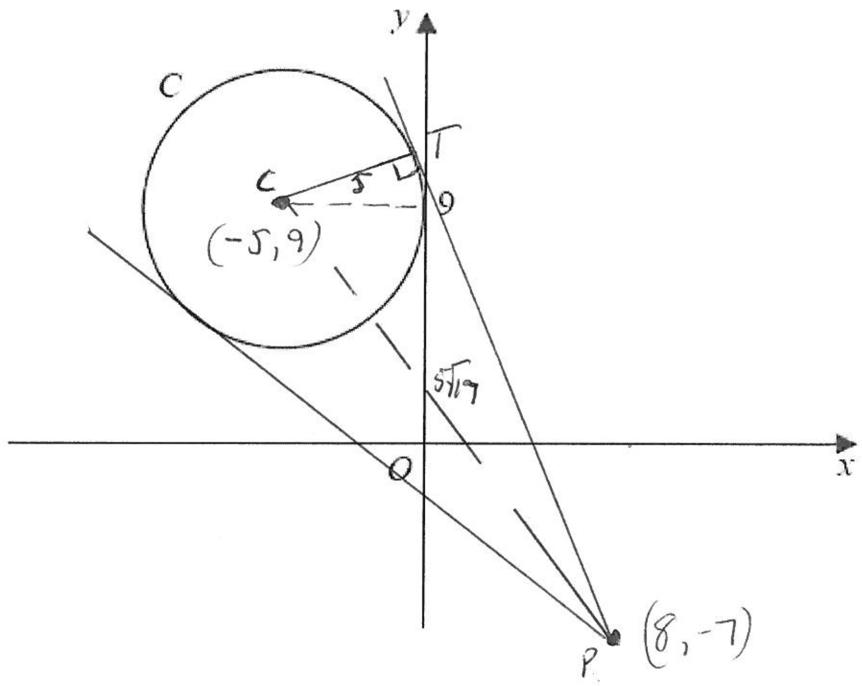
Equations of Circles

Things to remember:

8. The general equation of a circle is $(x - a)^2 + (y - b)^2 = r^2$, where (a, b) is the centre and r is the radius.
9. To calculate the equation of the tangent:
    1. Calculate the gradient of the radius of the circle.
    2. Calculate the gradient of the tangent of the circle.
    3. Substitute the given coordinate and the gradient of the tangent into $y = mx + c$ to calculate the y-intercept.

Questions:

1. The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in the diagram.
    (a) Write down an equation for the circle C, that is shown in the diagram.



r^2

$(x+5)^2 + (y-9)^2 = 25$

$C\ (-5, 9)$

$(x+5)^2 + (y-9)^2 = 25$ ....

(3)

A line through the point P(8, − 7) is a tangent to the circle C at the point T.

(b) Find the length of PT.

First, sketch this. Always sketch

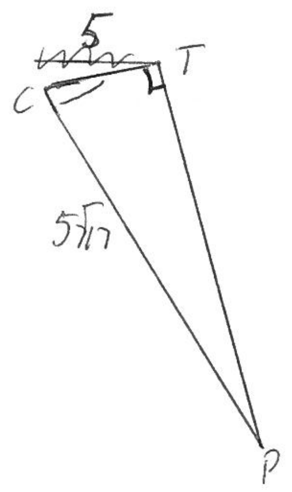
$\sqrt{(8 - -5)^2 + (-7 - 9)^2} = 5\sqrt{17} = PC$

Pythagoras: $(PT)^2 + 5^2 = (PC)^2$

$(PT)^2 + 25 = 425$

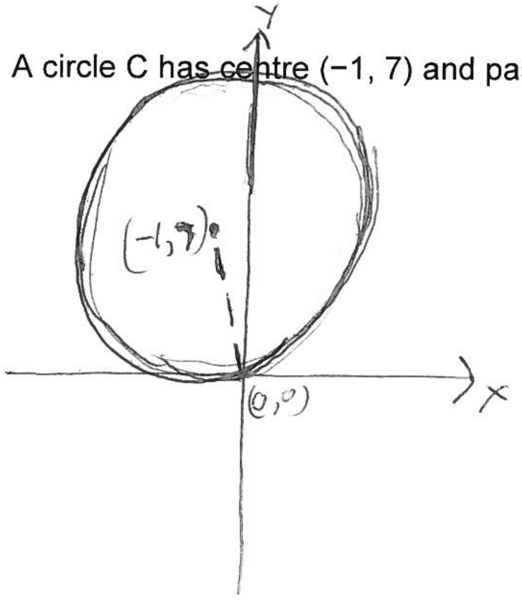
$(PT)^2 = 400$

$PT = 20$



(3)
(Total 6 marks)

2. A circle C has centre (−1, 7) and passes through the point (0, 0). Find an equation for C.



Distance from (-1, 7) to (0, 0)

= radius

$= \sqrt{(-1-0)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$

$(x - -1)^2 + (y-7)^2 = (\sqrt{50})^2$

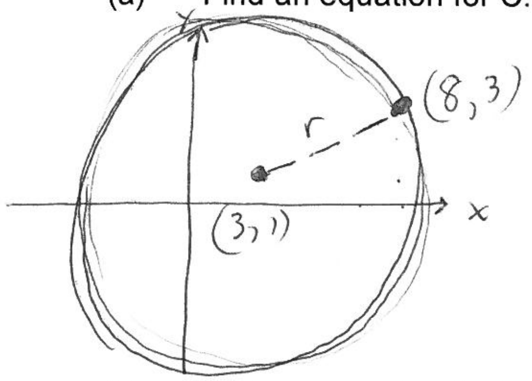
$(x+1)^2 + (y-7)^2 = 5$

$(x+1)^2 + (y-7)^2 = 50$

(Total 4 marks)

3. The circle C has centre (3, 1) and passes through the point P(8, 3).

(a) Find an equation for C.



$\sqrt{(8-3)^2 + (3-1)^2} = \sqrt{25+4} = \sqrt{29}$

$(x-3)^2 + (y-1)^2 = 29$

$(x-3)^2 + (y-1)^2 = 29$

(4)

(b) Find an equation for the tangent to C at P.

the line crossing (3, 1) and (8,3) will be $\perp$ to tangent at (8, 3)

① calculate $m = \frac{3-1}{8-3} = \frac{2}{5}$

② $y = mx + c$ $(3, 1)$ $m = \frac{2}{5}$

$1 = \frac{2}{5}(3) + c$ $c = -\frac{1}{5}$

$y = \frac{2}{5}x - \frac{1}{5}$

③ $m_{tan} = -\frac{5}{2}$

④ $y = mx + c$ $(8, 3)$ $m = -5/2$

$3 = -\frac{5}{2}(8) + c$ $c = 23$

$y = -5/2\, x + 23$

$y = -5/2\, x + 23$

(5)

(Total 9 marks)

Quadratic and Other Sequences

Things to remember:

- To calculate the nth term of a quadratic sequence:
  1. Calculate the first difference.
  2. Calculate the second difference.
  3. How many n^2s?
  4. Subtract.
  5. Calculate the nth term of the difference.
  6. Write the quadratic nth term.

Questions:

1. Here are the first 5 terms of a quadratic sequence.

   1 3 7 13 21

   Find an expression, in terms of n, for the nth term of this quadratic sequence.

1 3 7 13 21

2 4 6 8

2 2 2

$\times\frac{1}{2}$

$1n^2$

1 4 9 16 25

1 0 −1 −2 −3 −4

−1 −1 −1 −1

$-n+1$

n^2-n+1

n^2-n+1 ....

(Total for question is 3 marks)

2. Here are the first six terms of a Fibonacci sequence.

   1 1 2 3 5 8

   The rule to continue a Fibonacci sequence is,

   the next term in the sequence is the sum of the two previous terms.

   (a) Find the 9th term of this sequence.

1 1 2 3 5 8 13 21 34

34 ....

(1)

The first three terms of a different Fibonacci sequence are

a b $a+b$

(b) Show that the 6th term of this sequence is $3a + 5b$

a b $a+b$ $b+a+b$ $a+b+b+a+b$ $b+a+b+a+b+b+a+b$

$= 3a+5b$

or

a b $a+b$ $a+2b$ $2a+3b$ $3a+5b$

(2)

Given that the 3rd term is 7 and the 6th term is 29,
(c) find the value of *a* and the value of *b*.

$(\times 3)$ $a+b=7$

$3a+5b=29$

sim. eqs

$3a+5b=29$

$-(3a+3b=21)$

$2b=8$

$b=4$

$2b=8$

$b=4$

check in both eq^s.

$a+4=7$ $\quad a=3$

$3a+5(4)=29$

$3a=9$

$a=3$

$a =$ ....3........

$b =$ ....4........

(3)

(Total for question = 6 marks)

3. Here are the first five terms of a sequence.

2 8 18 32 50

(a) Find the next term of this sequence.

$+6 \quad +10 \quad +14 \quad +18 \quad +22$

can work out quadratic sequence long way also

........72........

(1)

The *n*th term of a different sequence is $3n^2 - 10$

(b) Work out the 5th term of this sequence.

$3(5)^2-10$

$=3(25)-10$

$=75-10$

$=65$

........65........

(1)

(Total for question = 2 marks)

4. Here are the first five terms of an arithmetic sequence.

1 5 9 13 17

(a) Write down an expression, in terms of *n*, for the *n*th term of this sequence.

-3 1 5 9 13 17

+4 +4 +4 +4

$4n-3$

(2)

The *n*th term of a different number sequence is $3n^2 + 7$

(b) Find the 10th term of this sequence.

$3(10)^2 + 7$

$= 3(100) + 7$

$= 300 + 7$

$= 307$

307

(2)

(Total for Question is 4 marks)

Completing the Square

Things to remember:

- To complete the square:
    1. Halve the coefficient of x.
    2. Put in brackets with the x and square the brackets.
    3. Subtract the half-coefficient squared.
    4. Don't forget the constant on the end!
    5. Simply.
- For $(x - p)^2 + q = 0$, the turning point is (p, q).

Questions:

1. (i) Sketch the graph of $f(x) = x^2 - 5x + 10$, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

$f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 10$

$= \left(x - \frac{5}{2}\right)^2 + \frac{15}{4}$

$\left(\frac{5}{2}, \frac{15}{4}\right)$ vertex

$f(x) = \left(0 - \frac{5}{2}\right)^2 + \frac{15}{4}$

$f(x) = \frac{40}{4} = 10$

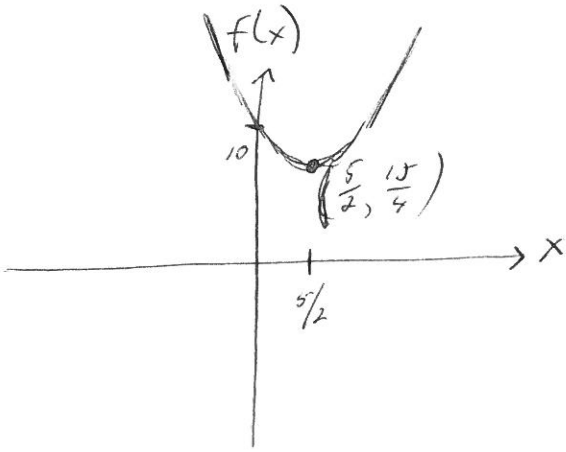
y-int

$\left(x - \frac{5}{2}\right)^2 + \frac{15}{4} = 0$

$\left(x - \frac{5}{2}\right)^2 = -\frac{15}{4}$

$x - \frac{5}{2} = \pm\sqrt{-\frac{15}{4}}$

no real roots



(ii) Hence, or otherwise, determine whether $f(x + 2) - 3 = 0$ has any real roots. Give reasons for your answer.

consider the transformations

$f(x+2) - 3$

translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

vertex: $f(x)$ was $\frac{15}{4}$

after $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ vertex $f(x) = \frac{15}{4} - 3 = \frac{3}{4}$

(Total for question = 6 marks)

still no real roots as $\frac{3}{4} > 0$

2. (a) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

$2(x^2 + 8x + \frac{35}{2})$

$2((x+4)^2 - 16 + \frac{35}{2})$

$2((x+4)^2 + \frac{3}{2})$

$2(x+4)^2 + 3$

$2(x+4)^2 + 3$ ....

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

vertex can be taken from CTS form

$(-4, 3)$

$(-4, 3)$ ....

(1)

(Total for question = 4 marks)

3. The expression $\cancel{x2}\ x^2 - 8x + 21$ can be written in the form $(x - a)\cancel{2}^2 + b$ for all values of x.
(a) Find the value of a and the value of b.

$(x-4)^2 - 16 + 21$

$(x-4)^2 + 5$

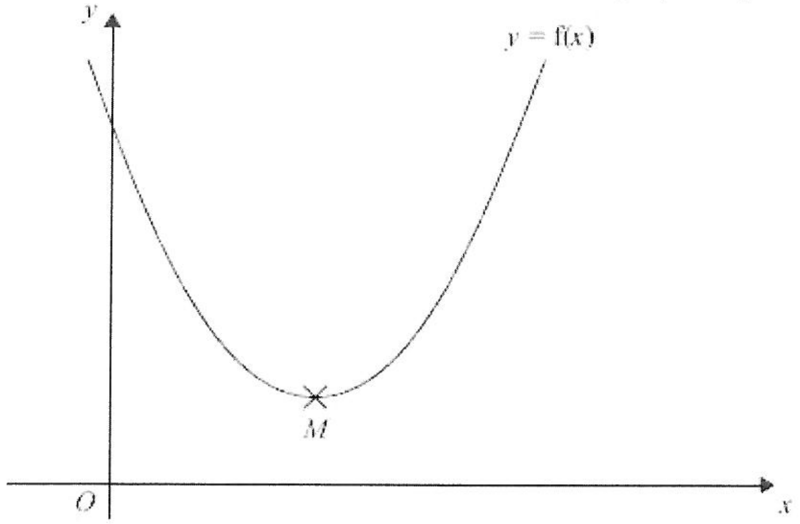
take care $a \neq -\cancel{4}$

$a =$ ......4......

$b =$ ......5......

(3)

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 8x + 21$
The diagram shows part of a sketch of the graph of $y = f(x)$.



The minimum point of the curve is M.
(b) Write down the coordinates of M.

$(4, 5)$ ....

(1)
(Total for Question is 4 marks)

$f(x) = (x-4)^2 + 5$

vertex $(4, 5)$

Inverse and Composite Functions

Things to remember:

- y = f(x) means that y is a function of x.
- f(a) means the value of x is a, so substitute x with a.
- The graph of the inverse is the reflection of the graph in the line y = x
- We find the inverse function by putting the original function equal to y and rearranging to make x the subject. $f^{-1}(x)$
- We use the notation ~~f-1(x)~~ for the inverse function.
- When a function is followed by another, the result is a composite function.
- fg(x) means do g first, followed by f.

Questions:

1. The functions f and g are such that

$$f(x) = 1 - 5x \quad \text{and} \quad g(x) = 1 + 5x$$

(a) Show that gf(1) = − 19

$$gf(x) = \underbrace{1 + 5\overbrace{(1-5x)}^{f(x)}}_{g(x)} = 1 + 5 - 25x = 6 - 25x$$

$$gf(1) = 6 - 25(1) = -19$$

(2)

(b) Prove that $f^{-1}(x) + g^{-1}(x) = 0$ for all values of x.

$$f(x) = 1 - 5x$$
$$y = 1 - 5x$$
$$5x = -y + 1$$
$$x = \frac{-y+1}{5}$$
$$y = \frac{-x+1}{5}$$
$$f^{-1}(x) = \frac{-x+1}{5}$$

$$g(x) = 1 + 5x$$
$$y = 1 + 5x$$
$$5x = y - 1$$
$$x = \frac{y-1}{5}$$
$$y = \frac{x-1}{5}$$
$$g^{-1}(x) = \frac{x-1}{5}$$

$$f^{-1}(x) + g^{-1}(x)$$
$$\frac{-x+1}{5} + \frac{x-1}{5}$$
$$= \frac{-x+1+x-1}{5}$$
$$= \frac{0}{5}$$
$$= 0$$

(3)

(Total for question = 5 marks)

2. The function f is such that

$f(x) = 4x - 1$

(a) Find $f^{-1}(x)$

$f(x) = 4x - 1$

$y = 4x - 1$

$4x = y + 1$

$x = \frac{y+1}{4}$

$y = \frac{x+1}{4}$

$f^{-1}(x) = \frac{x+1}{4}$

$f^{-1}(x) =$ .... $\frac{x+1}{4}$ ....

(2)

The function g is such that

$g(x) = kx^2$ where k is a constant.

Given that $fg(2) = 12$

(b) work out the value of k

$fg(x) = 4(kx^2) - 1$

$= 4kx^2 - 1$

$4k(2)^2 - 1 = 12$

$16k - 1 = 12$

$16k = 13$

$k = 13/16$

$k =$ .... $13/16$ ....

(2)

(Total for question = 4 marks)

3. The functions f and g are such that

$f(x) = 3(x - 4)$ and $g(x) = \frac{x}{5} + 1$

(a) Find the value of f(10)

$f(10) = 3(10 - 4)$

$= 3(6)$

$= 18$

.... 18 ....

(1)

(b) Find $g^{-1}(x)$

$g(x) = \frac{x}{5} + 1$

$y = \frac{x}{5} + 1$

$5y = x + 5$

$x = 5y - 5$

$y = 5x - 5$

$g^{-1}(x) = 5x - 5$

$g^{-1}(x) =$ .... $5x - 5$ ....

(2)

(c) Show that $ff(x) = 9x - 48$

$f(f(x)) = 3((3(x-4)) - 4)$

$= 9(x-4) - 12$

$= 9x - 36 - 12$

$= 9x - 48$

(2)

(Total for question = 5 marks)

4. $f(x) = 3x^2 - 2x - 8$
Express $f(x + 2)$ in the form $ax^2 + bx$

$3(x+2)^2 - 2(x+2) - 8$

$= 3(x^2 + 4x + 4) - 2x - 4 - 8$

$= 3x^2 + 12x + 12 - 2x - 12$

gather like terms

$= 3x^2 + 10x$

$3x^2 + 10x$

(Total for question is 3 marks)

Expanding more than two binomials

Things to remember:

- Start by expanding two pair of brackets using the grid or FOIL method.
- Then expand the third set of brackets.
- Use columns to keep x^3, x^2 etc in line to help with addition.

Questions:

1. Show that

$$(x - 1)(x + 2)(x - 4) = x^3 - 3x^2 - 6x + 8$$

for all values of x.

$(x-1)(x+2)$

$= x^2 + 2x - x - 2$

$= x^2 + x - 2$

$(x-4)(x^2 + x - 2)$

$= x^3 + x^2 - 2x$

$-4x^2 - 4x + 8$

$x^3 - 3x^2 - 6x + 8$

....

(Total for question is 3 marks)

2. Show that

$$(3x - 1)(x + 5)(4x - 3) = 12x^3 + 47x^2 - 62x + 15$$

for all values of x.

$(3x-1)(x+5)$

$= 3x^2 + 15x - x - 5$

$= 3x^2 + 14x - 5$

$(4x-3)(3x^2 + 14x - 5)$

$= 12x^3 + 56x^2 - 20x$

$-9x^2 - 42x + 15$

$12x^3 + 47x^2 - 62x + 15$

....

(Total for question is 3 marks)

3. Show that

$$(x - 3)(2x + 1)(x + 3) = 2x^3 + x^2 - 18x - 9$$

for all values of x.

$(x-3)(2x+1)$

$= 2x^2 + x - 6x - 3$

$= 2x^2 - 5x - 3$

$(x+3)(2x^2 - 5x - 3)$

$= 2x^3 - 5x^2 - 3x$

$\quad + 6x^2 - 15x - 9$

$2x^3 + x^2 - 18x - 9$

....

(Total for question is 3 marks)

4. $(2x + 1)(x + 6)(x - 4) = 2x^3 + ax^2 + bx - 24$

for all values of x, where a and b are integers.

Calculate the values of a and b.

$(2x+1)(x+6)$

$= 2x^2 + 12x + x + 6$

$= 2x^2 + 13x + 6$

$(x-4)(2x^2 + 13x + 6)$

$= 2x^3 + 13x^2 + 6x$

$\quad - 8x^2 - 52x - 24$

$2x^3 + 5x^2 - 46x - 24$

a = 5

b = -46

(Total for question is 4 marks)

Nonlinear Simultaneous Equations

Things to remember:

1. Substitute the linear equation into the nonlinear equation.
2. Rearrange so it equals 0.
3. Factorise and solve for the first variable (remember there will be two solutions).
4. Substitute the first solutions to solve for the second variable.
5. Express the solution as a pair of coordinate where the graphs intersect.

Questions:

1. Solve the equations

$x^2 + y^2 = 36$

$x = 2y + 6 \longleftrightarrow y = \frac{1}{2}x - 3$

$(2y+6)^2 + y^2 = 36$

$4y^2 + 24y + 36 + y^2 = 36$

$\quad -36 \qquad -36$

$5y^2 + 24y = 0$

$y(5y + 24) = 0$

$y = 0 \qquad y = \frac{-24}{5}$

$y = 0$

$x^2 + 0^2 = 36$

$x = \pm 6$

$(6,0)(-6,0)$

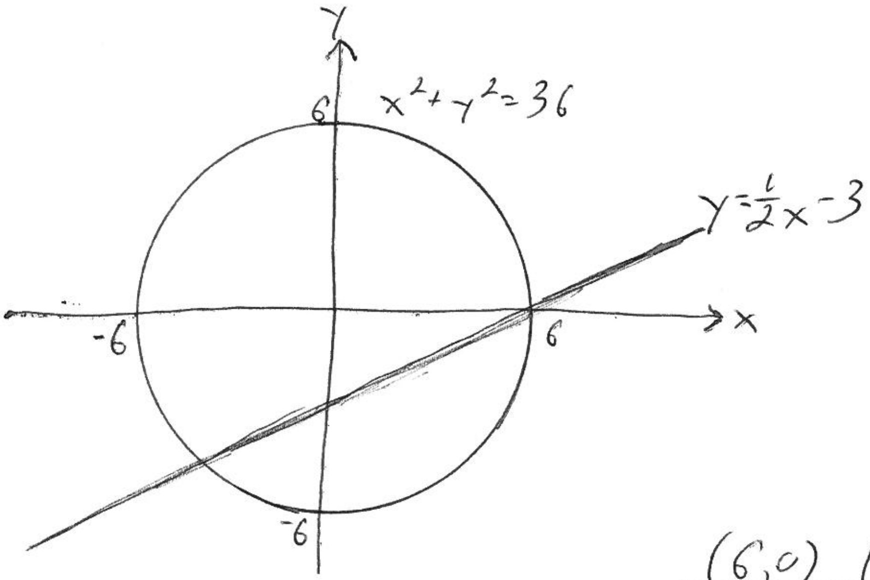
$y = \frac{-24}{5} = -4\frac{4}{5}$

$x = 2\left(\frac{-24}{5}\right) + 6$

$= \frac{-48}{5} + \frac{30}{5}$

$= \frac{-18}{5} = -3\frac{3}{5}$

$\left(-3\frac{3}{5}, -4\frac{4}{5}\right)$



$(6,0)\ \left(-3\frac{3}{5}, -4\frac{4}{5}\right)$

(Total for Question is 5 marks)

(sketches enhance understanding)

3. Solve the simultaneous equations

$$x^2 + y^2 = 25$$
$$y = 2x + 5$$

$x^2 + (2x+5)^2 = 25$

$x^2 + 4x^2 + 20x + \cancel{25} = \cancel{25}$

$x^2 + 4x^2 + 20x = 0$

$5x^2 + 20x = 0$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \quad x = -4$

$x = 0$

$y^2 = 25$

$y = \pm 5$

$(0,5)\ (0,-5)$

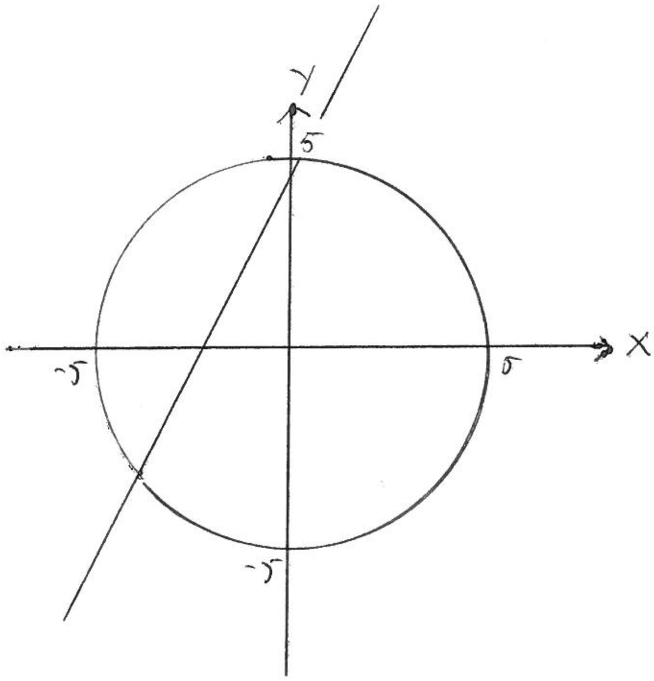
$x = -4$

$16 + y^2 = 25$

$y^2 = 9$

$y = \pm 3$

$(-4,3)\ (-4,-3)$



(again, a sketch gives the complete picture and allows you to see which solutions are relevant for the question asked)

$x =$ ..... 0 ..... and $y =$ ..... 5 .....
or
$x =$ ..... −4 ..... and $y =$ ..... −3 .....

(Total for Question is 6 marks)

for sketch

4. Solve the simultaneous equations $x^2 + y^2 = 9$

$x + y = 2 \longleftrightarrow x = -y+2 \longleftrightarrow y = -x+2$

Give your answers correct to 2 decimal places.

$(-y+2)^2 + y^2 = 9$

$y^2 - 4y + 4 + y^2 = 9$

$2y^2 - 4y - 5 = 0$

~~$(2y\quad)(y\quad)$~~

$2(y^2 - 2y - \frac{5}{2}) = 0$

$y^2 - 2y - \frac{5}{2} = 0$

$(y-1)^2 - 1 - \frac{5}{2} = 0$

$(y-1)^2 = \frac{7}{2}$

$y - 1 = \pm\sqrt{\frac{7}{2}}$

$y = 1 \pm \sqrt{\frac{7}{2}}$

$y = 1 + \sqrt{\frac{7}{2}}$ $\quad$ $y = 1 - \sqrt{\frac{7}{2}}$

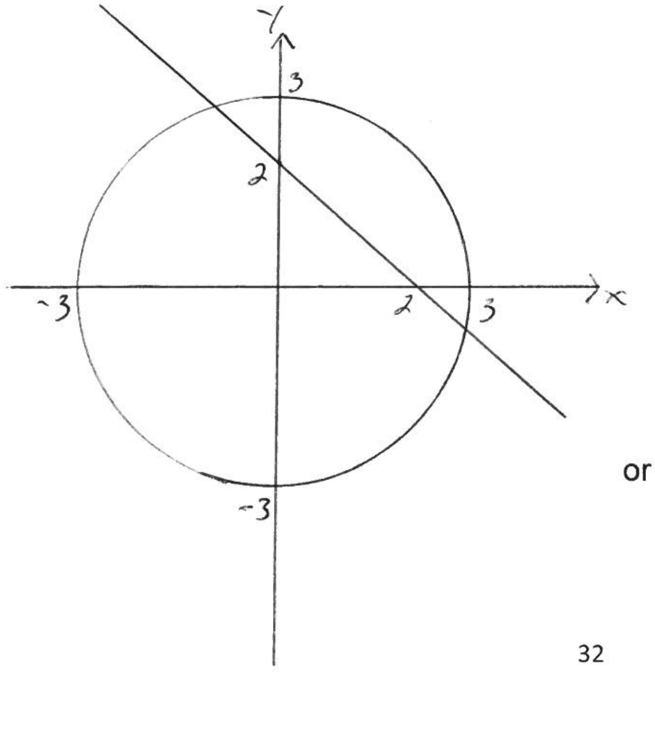
$x + 1 + \sqrt{\frac{7}{2}} = 2$ $\quad$ $x + 1 - \sqrt{\frac{7}{2}} = 2$

$x = 1 - \sqrt{\frac{7}{2}}$ $\quad$ $x = 1 + \sqrt{\frac{7}{2}}$

$\left(1 - \sqrt{\frac{7}{2}}, 1 + \sqrt{\frac{7}{2}}\right)$ $\quad$ $\left(1 + \sqrt{\frac{7}{2}}, 1 - \sqrt{\frac{7}{2}}\right)$

$\sqrt{\frac{7}{2}} = \sqrt{3.5}$ = a bit less than 2 (≈ 1.9)

$\underbrace{(-0.9, 2.9) \quad (2.9, -0.9)}_{\text{roughly (for sketch)}}$



$x = 1 - \sqrt{\frac{7}{2}}$ .... $y = 1 + \sqrt{\frac{7}{2}}$ ....

or $x = 1 + \sqrt{\frac{7}{2}}$ .... $y = 1 - \sqrt{\frac{7}{2}}$ ....

(Total for Question is 6 marks)

5. Solve algebraically the simultaneous equations

$x^2 + y^2 = 25$

$y - 2x = 5 \longleftrightarrow y = 2x + 5$

$x^2 + (2x+5)^2 = 25$

$x^2 + 4x^2 + 20x \cancel{+25} = \cancel{25}$

$x^2 + 4x^2 + 20x = 0$

$5x^2 + 20x = 0$

$x^2 + 4x = 0$

Ah, this is looking familiar...

... see Q 3

....

(Total for Question is 5 marks)

Solving Quadratic Inequalities

Things to remember:

- Start by solving the quadratic to find the values of x, then sketch the graph to determine the inequality.

Questions:

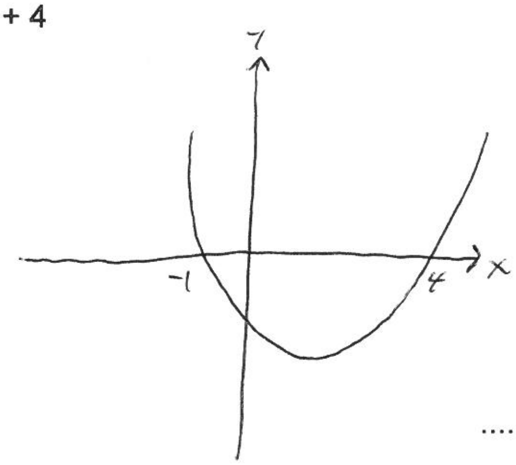
1. Solve $x^2 > 3x + 4$

$x^2 - 3x - 4 > 0$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$\underbrace{x = 4 \quad x = -1}_{\text{critical values}}$



$x < -1$ or $x > 4$

(Total for question = 3 marks)

2. Solve the inequality $x^2 > 3(x + 6)$

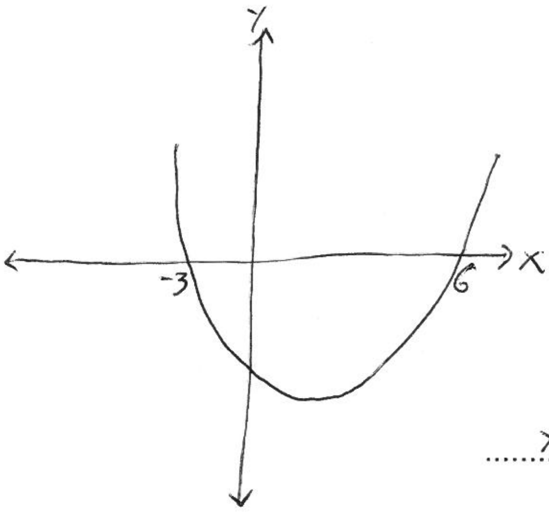
$x^2 > 3x + 18$

$x^2 - 3x - 18 > 0$

$x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$\underbrace{x = 6 \quad x = -3}_{\text{critical values}}$



$x < -3$ or $x > 6$

(Total for question = 4 marks)

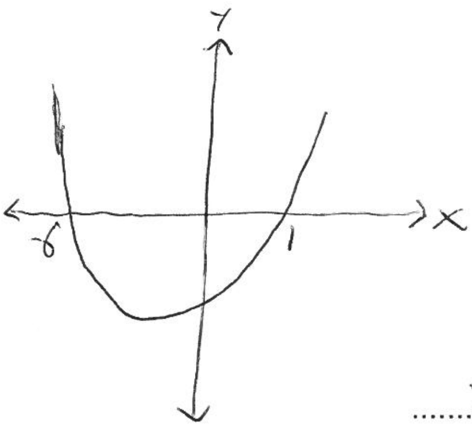
3. Solve the inequality $x^2 + 5x > 6$

$x^2 + 5x - 6 > 0$

$x^2 + 5x - 6 = 0$

$(x + 6)(x - 1) = 0$

$\underbrace{x = -6 \quad x = 1}_{\text{critical values}}$



$x < -6$ or $x > 1$

(Total for question = 3 marks)

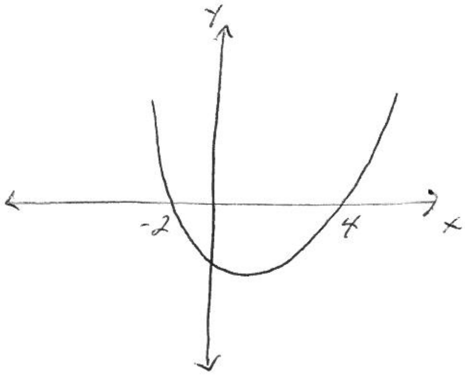
4. Solve the inequality $x^2 - 2x + 8 < 0$

$\uparrow$

$x^2 - 2x + 8 = 0$

$(x-4)(x+2) = 0$

$\underbrace{x = 4 \quad x = -2}_{\text{critical values}}$



$-2 < x < 4$

(Total for question = 3 marks)

5. Solve the inequality $x^2 - x \geq 12$

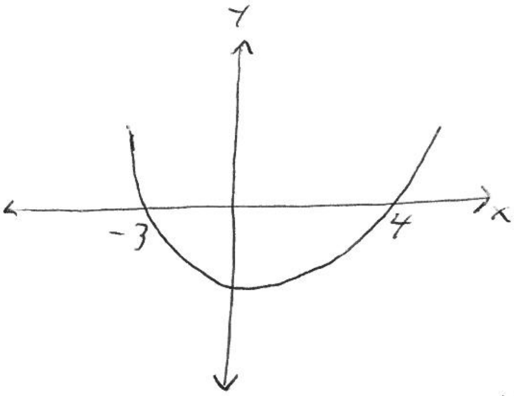
$x^2 - x - 12 \geq 0$

$\uparrow$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$\underbrace{x = 4 \quad x = -3}_{\text{critical values}}$



$x \leq -3$ or $x \geq 4$

(Total for question = 3 marks)

6. Solve the inequality $x^2 \leq 4(2x + 5)$

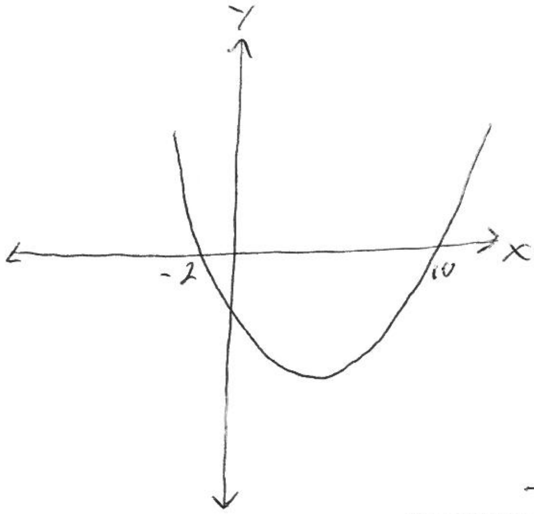
$x^2 \leq 8x + 20$

$x^2 - 8x - 20 \leq 0$

$\uparrow$

$(x-10)(x+2) = 0$

$\underbrace{x = 10 \quad x = -2}_{\text{critical values}}$

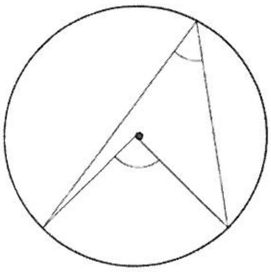


$-2 \leq x \leq 10$

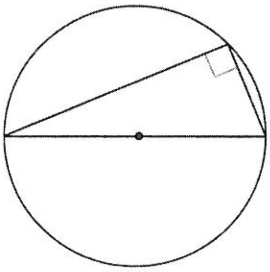
(Total for question = 4 marks)

Circle theorems

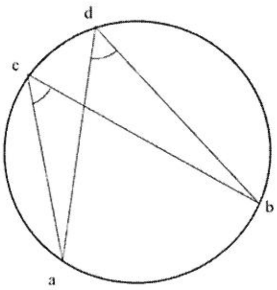
Things to remember:



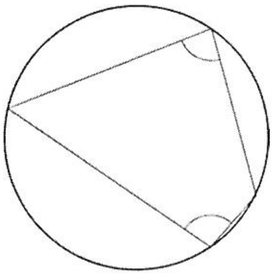
The angle at the centre is twice the angle at the circumference.



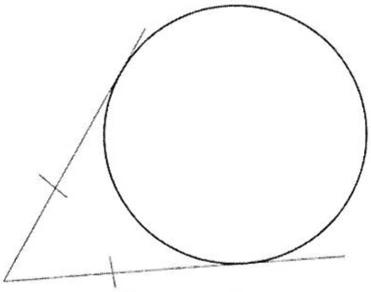
The angle in a semi-circle is 90°.



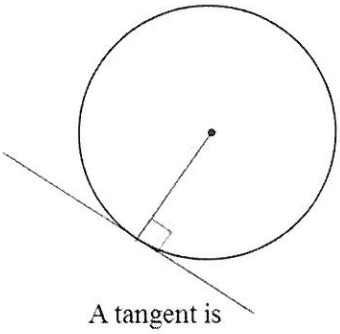
Angles subtended by the same arc are equal.



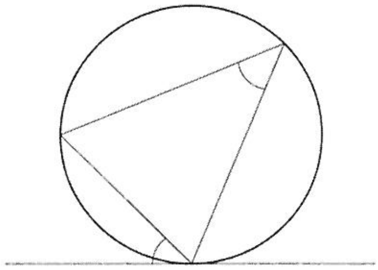
Opposite angles in a cyclic quadrilateral sum to 180°.



Tangents from a point are equal.



A tangent is perpendicular to a radius.



Angles in alternate segments are equal.

Questions:

1.

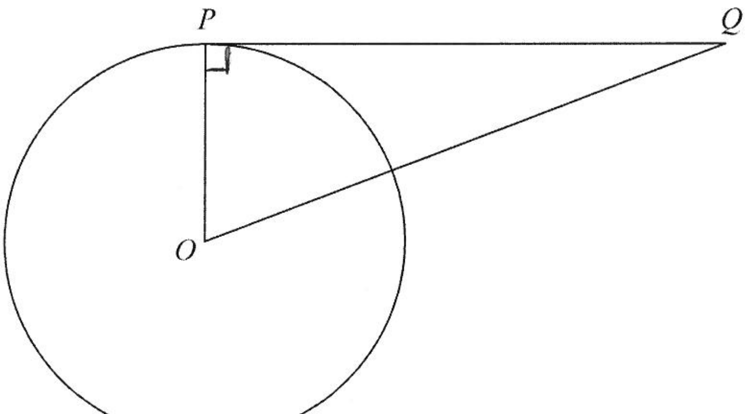


Diagram **NOT** accurately drawn

P is a point on the circumference of the circle, centre *O*.

PQ is a tangent to the circle.

(i) Write down the size of angle *OPQ*. ....90°.... °

(ii) Give a reason for your answer.

....a tangent is perpendicular to a radius....

....

(Total 2 marks)

2.

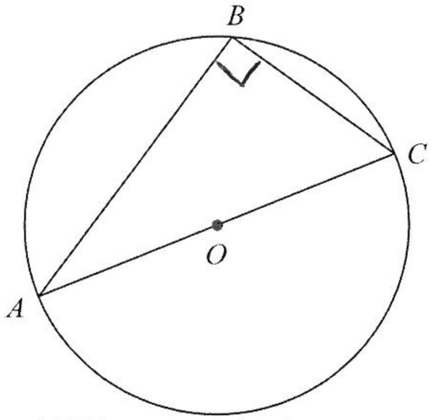


Diagram **NOT** accurately drawn

A, *B* and *C* are points on the circumference of a circle, centre *O*.

AC is a diameter of the circle.

(a) (i) Write down the size of angle *ABC*. 90° °

(ii) Give a reason for your answer.

The angle in a semicircle is 90°

(2)

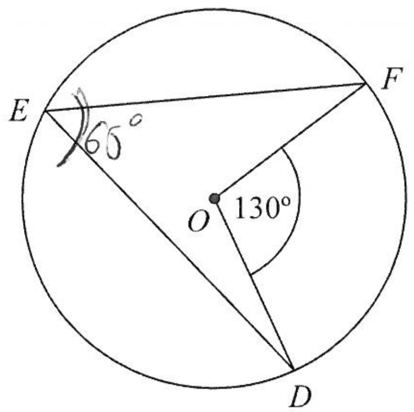


Diagram **NOT** accurately drawn

D, *E* and *F* are points on the circumference of a circle, centre *O*.

Angle *DOF* = 130°.

(b) (i) Work out the size of angle *DEF*. 65° °

(ii) Give a reason for your answer.

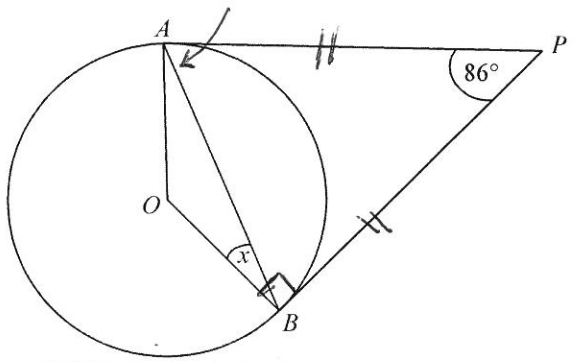
The angle at the centre is twice the angle at the circumference

(2)

(Total 4 marks)

$\frac{1}{2}(180-86) = 47°$

3.



$x = 90 - 47 = 43°$

Diagram **NOT** accurately drawn

A and *B* are points on the circumference of a circle, centre *O*.

PA and *PB* are tangents to the circle.

Angle *APB* is 86°.

Work out the size of the angle marked *x*.

..........43°.......... °

(Total 2 marks)

4.

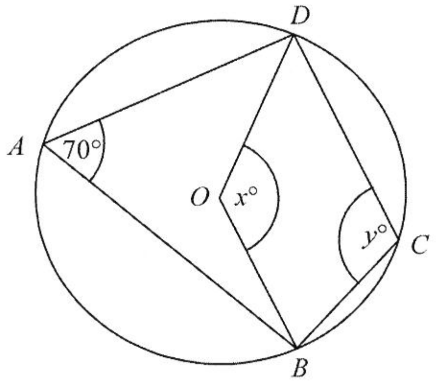


Diagram **NOT** accurately drawn

In the diagram, *A*, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Angle *BAD* = 70°.

Angle *BOD* = *x*°.

Angle *BCD* = *y*°.

(a) (i) Work out the value of *x*.

..........140°.......... °

(ii) Give a reason for your answer.

..........Angle at centre is twice the..........

..........angle at the circumference..........

(2)

(b) (i) Work out the value of *y*.

..........$180 - 70 = 110°$.......... °

(ii) Give a reason for your answer.

..........ABCD is a cyclic quadrilateral..........

..........opposite angles in a cyclic..........

quadrilateral sum to 180°

(2)

(Total 4 marks)

5.

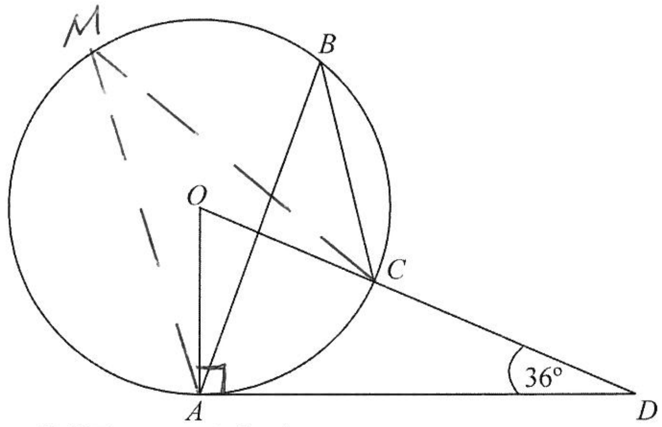


Diagram **NOT** accurately drawn

The diagram shows a circle centre *O*.

A, *B* and *C* are points on the circumference.

DCO is a straight line.

DA is a tangent to the circle.

Angle *ADO* = 36°

(a) Work out the size of angle *AOD*.

Σ angles in Δ radius meets tangent

180 - 90 - 36 = 54° °

(2)

(b) (i) Work out the size of angle *ABC*.

$\frac{1}{2}(54) = 27°$ °

(ii) Give a reason for your answer.

< AOC is angle at centre and so is 2x

angle at circumference (my < AMC)

(3)

(Total 5 marks)

<AMC = < ABC

angles subtended by the same arc

are equal

Vectors

Things to remember:

- Use the letter provided in the question.
- Going against the arrow is a negative.
- Vectors need to be written in bold or underlined.
- They can be manipulated similarly to algebra.

Questions:

1. The diagram shows a regular hexagon *ABCDEF* with centre *O*.

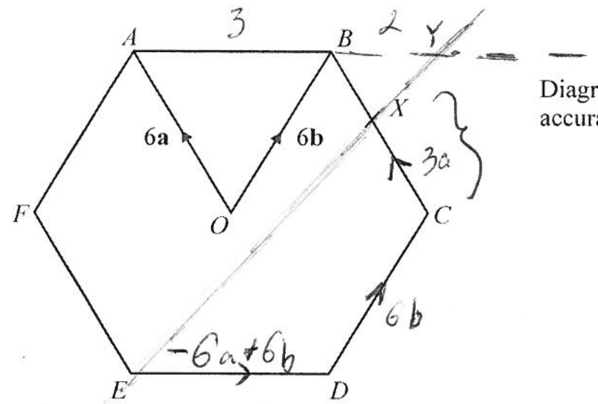


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

(a) Express in terms of **a** and/or **b**

(i) $\overrightarrow{AB}$, $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -6a + 6b$ ....$-6a+6b$........

(ii) $\overrightarrow{EF}$. $\overrightarrow{EF} = \overrightarrow{OA}$ ....$6a$........

(2)

X is the midpoint of *BC*.

(b) Express $\overrightarrow{EX}$ in terms of **a** and/or **b**

$\overrightarrow{EX} = -6a + 6b + 6b + 3a$

$= -3a + 12b$

........$-3a+12b$........

(2)

Y is the point on *AB* extended, such that *AB* : *BY* = 3:2

(c) Prove that *E*, *X* and *Y* lie on the same straight line.

$\overrightarrow{BY} = \frac{2}{3}(\overrightarrow{AB}) = \frac{2}{3}(-6a+6b) = -4a + 4b$

$\overrightarrow{XB} = 3a$, so $\overrightarrow{XY} = 3a - 4a + 4b = -a + 4b$

$\overrightarrow{EX} = -3a + 12b$
$\overrightarrow{XY} = -a + 4b$
scalar multiple so $\overrightarrow{EX}$ parallel $\overrightarrow{XY}$

(3)
(Total 7 marks)

$\overrightarrow{EX}$ ∥ $\overrightarrow{XY}$ with point X in common so E, X, Y co-linear

2. T is the point on PQ for which $PT : TQ = 2 : 1$.

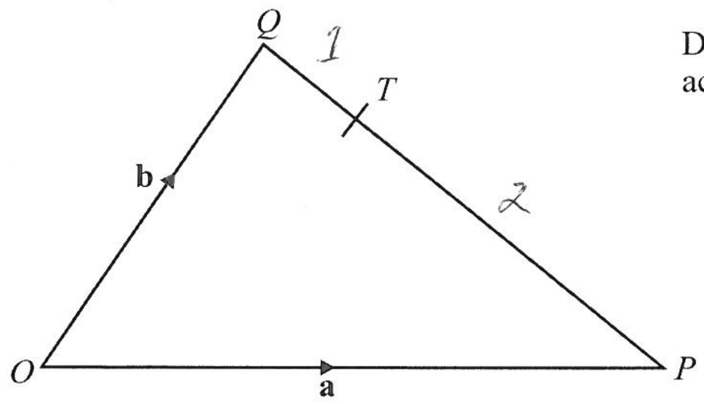


Diagram **NOT** accurately drawn

OPQ is a triangle.

$\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OQ} = \mathbf{b}$.

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{PQ}$.

$\overrightarrow{PQ}$ = $-a + b$

(1)

(b) Express $\overrightarrow{OT}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{OT} = \overrightarrow{OQ} + \overrightarrow{QT}$

$= b + \frac{1}{3}(\overrightarrow{QP})$

$= b + \frac{1}{3}(a - b)$

$= b + \frac{1}{3}a - \frac{1}{3}b$

$= \frac{1}{3}a + \frac{2}{3}b$

$\overrightarrow{OT}$ = $\frac{1}{3}a + \frac{2}{3}b$

(2)

(Total 3 marks)

3. *OABC* is a parallelogram.

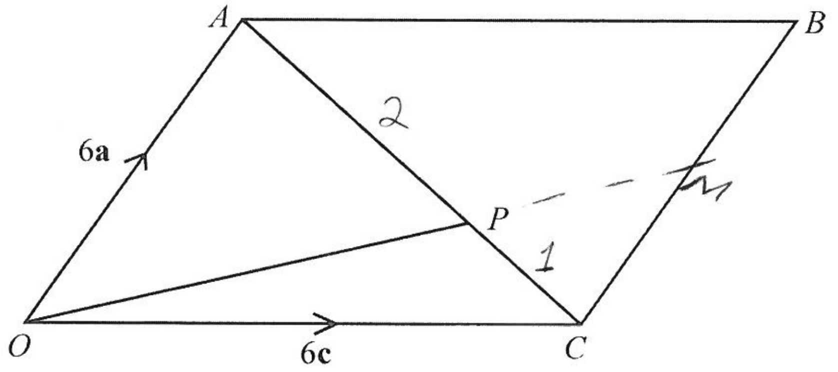


Diagram **NOT** accurately drawn

P is the point on *AC* such that $AP = \frac{2}{3}AC$.

$\overrightarrow{OA} = 6\mathbf{a}$. $\overrightarrow{OC} = 6\mathbf{c}$.

(a) Find the vector $\overrightarrow{OP}$.
Give your answer in terms of **a** and **c**.

$\overrightarrow{AC} = -6a + 6c$

$\overrightarrow{AP} = \frac{2}{3}(-6a + 6c) = -4a + 4c$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= 6a - 4a + 4c$

$= 2a + 4c$

$2a + 4c$

(3)

The midpoint of *CB* is *M*.

(b) Prove that *OPM* is a straight line.

calculate $\overrightarrow{PM}$

$\overrightarrow{PM} = \overrightarrow{PC} + \overrightarrow{CM}$

$-2a + 2c$ $\qquad \frac{1}{2}(6a)$

$3a$

$\overrightarrow{PM} = -2a + 2c + 3a$

$= a + 2c$

$\left.\begin{array}{l}\overrightarrow{OP} = 2a + 4C \\ \overrightarrow{PM} = a + 2c\end{array}\right\}$ scalar multiple so $\overrightarrow{OP} \parallel \overrightarrow{PM}$

$\overrightarrow{OP} \parallel \overrightarrow{PM}$ sharing point P so O, P, M co-linear

(2)
(Total 5 marks)

4. *OPQ* is a triangle.
R is the midpoint of *OP*.
S is the midpoint of *PQ*.
$\overrightarrow{OP}$ = **p** and $\overrightarrow{OQ}$ = **q**

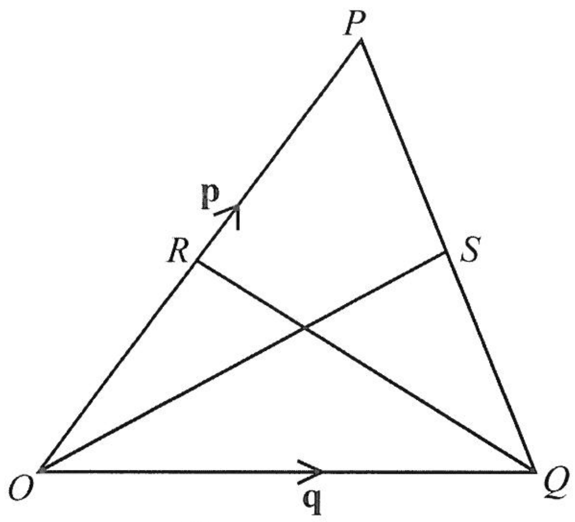


Diagram **NOT** accurately drawn

$\overrightarrow{PQ} = -p + q$
$\overrightarrow{PS} = \frac{1}{2}(\overrightarrow{PQ}) = -\frac{1}{2}p + \frac{1}{2}q$
$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$
$= p + (-\frac{1}{2}p + \frac{1}{2}q)$
$= \frac{1}{2}p + \frac{1}{2}q$

(i) Find $\overrightarrow{OS}$ in terms of **p** and **q**.

$\overrightarrow{OS}$ = $\frac{1}{2}p + \frac{1}{2}q$

(ii) Show that *RS* is parallel to *OQ*.

$\overrightarrow{RP} = \frac{1}{2}\overrightarrow{OP} = \frac{1}{2}p$
$\overrightarrow{RS} = \overrightarrow{RP} + \overrightarrow{PS}$
$= \frac{1}{2}p + (-\frac{1}{2}p + \frac{1}{2}q)$
$= \frac{1}{2}q$
$\overrightarrow{OQ} = q$

$\left.\begin{array}{l}\overrightarrow{RS} = \frac{1}{2}q \\ \overrightarrow{OQ} = q\end{array}\right\}$ scalar multiples

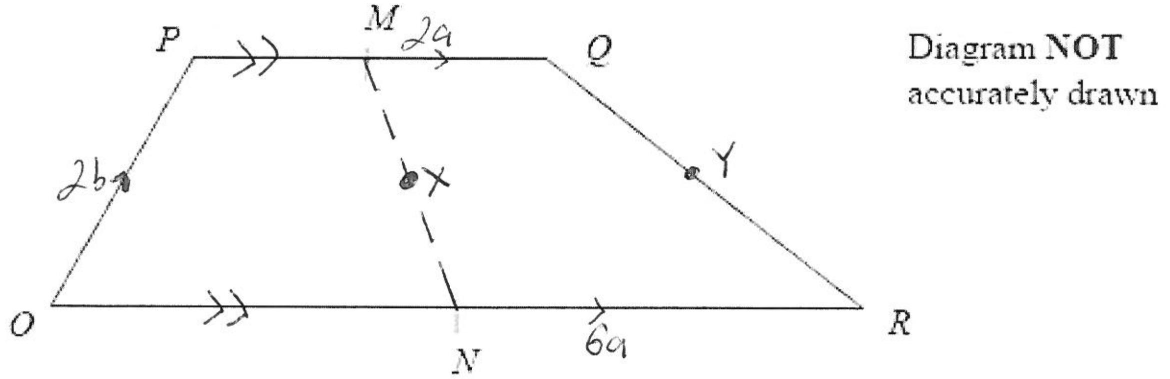
2 vectors that are scalar multiples are parallel

(Total 5 marks)

5. *OPQR* is a trapezium with *PQ* parallel to *OR*.

$\overrightarrow{OP} = 2\mathbf{b}$ $\quad \overrightarrow{PQ} = 2\mathbf{a}$ $\quad \overrightarrow{OR} = 6\mathbf{a}$

M is the midpoint of *PQ* and *N* is the midpoint of *OR*.



(a) Find the vector $\overrightarrow{MN}$ in terms of **a** and **b**.

$\overrightarrow{MP} = \frac{1}{2}\overrightarrow{QP} = -a$

$\overrightarrow{PO} = -2b$

$\overrightarrow{ON} = \frac{1}{2}\overrightarrow{OR} = 3a$

$\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PO} + \overrightarrow{ON}$

$= -a + -2b + 3a$

$= 2a - 2b$

$\overrightarrow{MN}$ = $2a - 2b$

(2)

X is the midpoint of *MN* and *Y* is the midpoint of *QR*.

(b) Prove that *XY* is parallel to *OR*.

$\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PO} + \overrightarrow{OR}$

$= -2a + -2b + 6a$

$= 4a - 2b$

$\overrightarrow{QY} = \frac{1}{2}QR = 2a - b$

$\cancel{\overrightarrow{XM}}$

$\overrightarrow{XM} = \frac{1}{2}NM = \frac{1}{2}(-2a + 2b)$

$= -a + b$

$\overrightarrow{XY} = \overrightarrow{XM} + \overrightarrow{MQ} + \overrightarrow{QY}$

$= -a + b + a + 2a - b$

$\left.\begin{array}{l} = 2a \\ \overrightarrow{OR} = 6a \end{array}\right\}$ scalar multiples

(2)

(Total 4 marks)

vectors that are scalar multiples of one another are parallel

6. *ABCD* is a straight line.

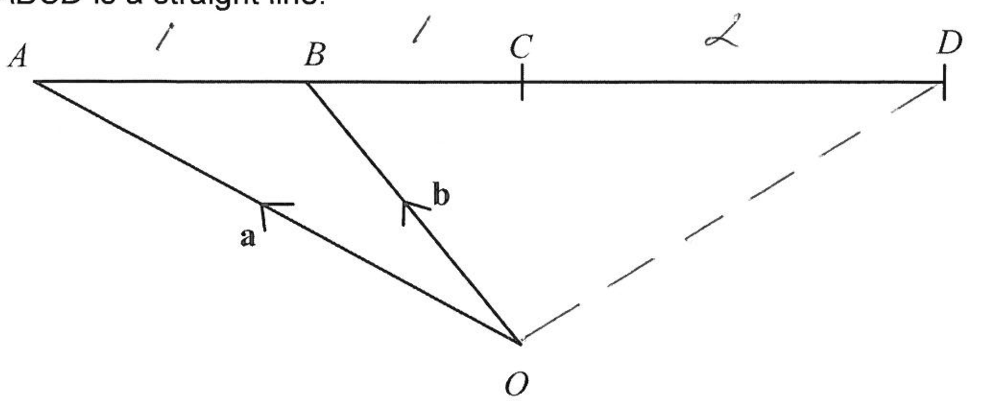


Diagram **NOT** accurately drawn

O is a point so that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

B is the midpoint of *AC*.

C is the midpoint of *AD*.

Express, in terms of **a** and **b**, the vectors

(i) $\overrightarrow{AC}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b$

$\overrightarrow{AC} = 2 \times \overrightarrow{AB} = 2(-a+b)$

$= -2a + 2b$

$-2a + 2b$ ....

(ii) $\overrightarrow{OD}$

$\overrightarrow{BC} = \overrightarrow{AB} = -a + b$

$\overrightarrow{BD} = 3 \times BC = 3(-a+b)$

$= -3a + 3b$

$-3a + 4b$ ....

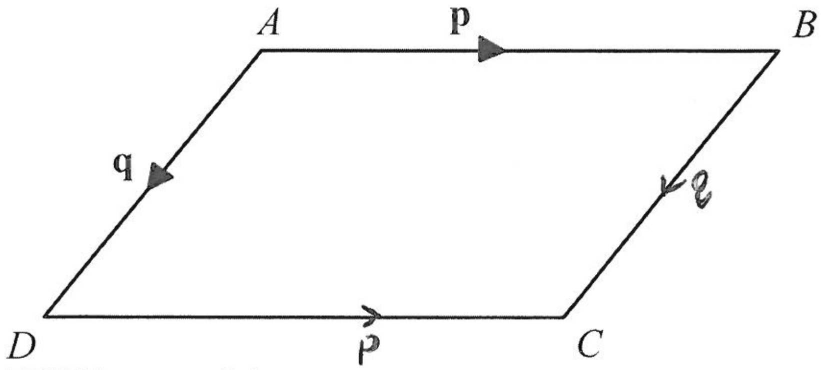
(Total 3 marks)

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= b + (-3a + 3b)$

$= -3a + 4b$

7. Diagram **NOT** accurately drawn



ABCD is a parallelogram.
AB is parallel to *DC*.
AD is parallel to *BC*.

$\vec{AB}$ = **p** $\quad \vec{DC} = p$
$\vec{AD}$ = **q** $\quad \vec{BC} = q$ } definition of parallelogram

$\vec{AC} = \vec{AD} + \vec{DC}$
$= q + p$

$\vec{BD} = \vec{BC} + \vec{CD}$
$= q + -p$
$= q - p$

(a) Express, in terms of p and q

(i) $\vec{AC}$

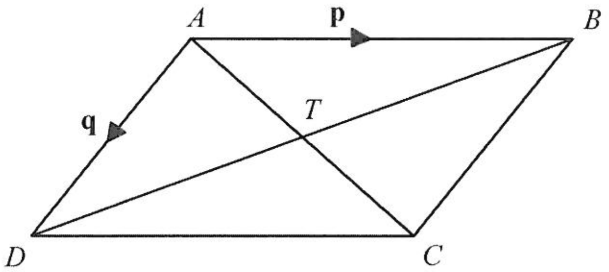
..........$q + p$..........

(ii) $\vec{BD}$

..........$q - p$..........

(2)

Diagram **NOT** accurately drawn



AC and *BD* are diagonals of parallelogram *ABCD*.
AC and *BD* intersect at *T*.

(b) Express $\vec{AT}$ in terms of **p** and **q.**

Property of a parallelogram: diagonals bisect each other

So, $\vec{AT} = \frac{1}{2}\vec{AC}$

$= \frac{1}{2}(q + p)$

$= \frac{1}{2}q + \frac{1}{2}p$

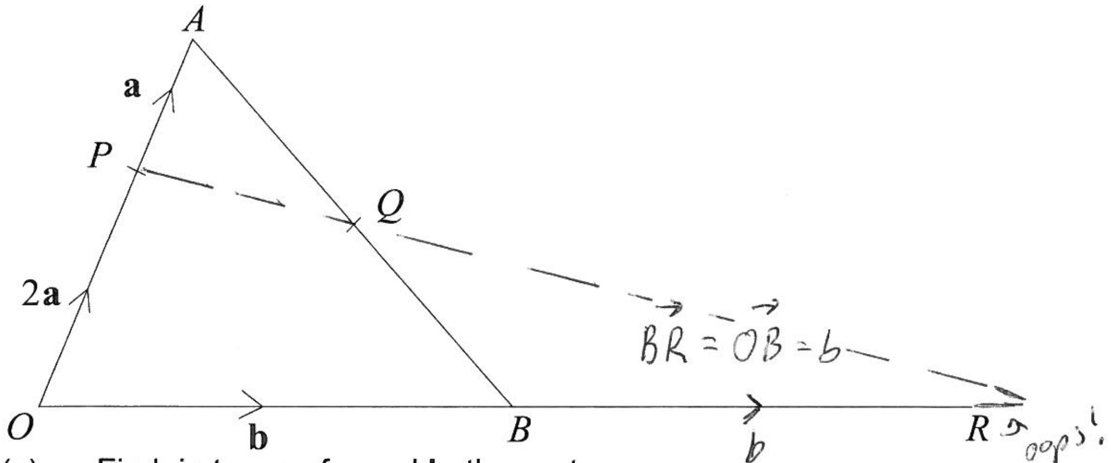
..........$\frac{1}{2}q + \frac{1}{2}p$..........

(1)
(Total 3 marks)

8. Diagram **NOT** accurately drawn

OAB is a triangle.
B is the midpoint of *OR*.
Q is the midpoint of *AB*.

$\overrightarrow{OP} = 2\mathbf{a} \quad \overrightarrow{PA} = \mathbf{a} \quad \overrightarrow{OB} = \mathbf{b}$



(a) Find, in terms of **a** and **b**, the vectors

(i) $\overrightarrow{AB}$, ... $-3a + b$

(ii) $\overrightarrow{PR}$, $= \overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BR}$
$= a + (-3a + b) + b$ .... $-2a + 2b$

(iii) $\overrightarrow{PQ}$. $\overrightarrow{AQ} = \frac{1}{2}\overrightarrow{AB} = -\frac{3}{2}a + \frac{1}{2}b$ .... $-\frac{1}{2}a + \frac{1}{2}b$
$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = a + (-\frac{3}{2}a + \frac{1}{2}b)$

(4)

(b) Hence explain why *PQR* is a straight line.

$\overrightarrow{QB} = \overrightarrow{AQ}$

$\overrightarrow{QR} = \overrightarrow{QB} + \overrightarrow{BR}$
$= (-\frac{3}{2}a + \frac{1}{2}b) + b$
$= -\frac{3}{2}a + \frac{3}{2}b$

$\overrightarrow{PQ} = -\frac{1}{2}a + \frac{1}{2}b$
$\overrightarrow{QR} = -\frac{3}{2}a + \frac{3}{2}b$
scalar multiples so parallel

parallel vectors which share a point (Q) form a straight line.

(2)

The length of *PQ* is 3 cm.

(c) Find the length of *PR*.

$\overrightarrow{QR} = 3 \times \overrightarrow{PQ}$

$\underbrace{-\frac{3}{2}a + \frac{3}{2}b \qquad -\frac{1}{2}a + \frac{1}{2}b}$

scalar multiple = 3

so, if PQ = 3 cm then QR = 9 cm and PR = 12 cm

.... 12 cm

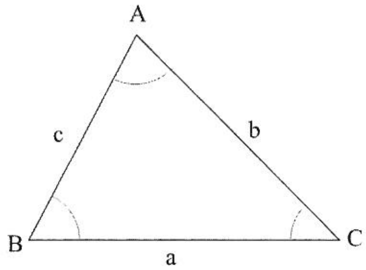
(1)

(Total 7 marks)

Sine and Cosine Rules

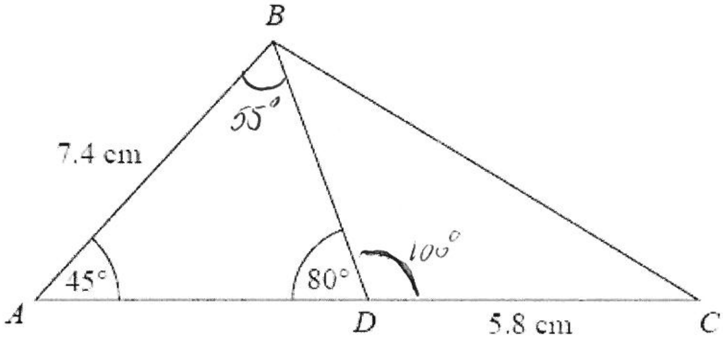
Things to remember:

- For any triangle ABC, $a^2 = b^2 + c^2 - 2bc \cos A$
- For any triangle ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
- For any triangle ABC, area = ½ a b sinC



Questions:

1. Diagram **NOT** accurately drawn
ABC is a triangle.
D is a point on *AC*.
Angle *BAD* = 45°
Angle *ADB* = 80°
AB = 7.4 cm
DC = 5.8 cm
Work out the length of *BC*.
Give your answer correct to 3 significant figures.



$\frac{DB}{\sin 45} = \frac{7.4}{\sin 80}$

$DB = \frac{7.4 \sin 45}{\sin 80}$

$= 5.313311319$

using $\boxed{ANS}$ this never leaves my calculator

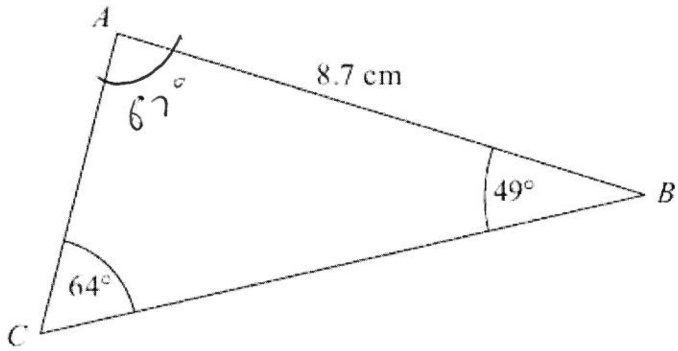
$(BC)^2 = (5.31\ldots)^2 + 5.8^2 - 2(5.31\ldots)(5.8)\cos 100$

$BC = \sqrt{\quad ''}$

8.52 (3 sf) cm

(Total for question = 5 marks)

2. Diagram **NOT** accurately drawn
ABC is a triangle.
AB = 8.7 cm.
Angle *ABC* = 49°.
Angle *ACB* = 64°.
Calculate the area of triangle *ABC*.
Give your answer correct to 3 significant figures.



$\frac{8.7}{\sin 64°} = \frac{AC}{\sin 49°}$

$AC = \frac{8.7 \sin 49}{\sin 64}$

$= 7.305314688$

again, use $\boxed{ANS}$

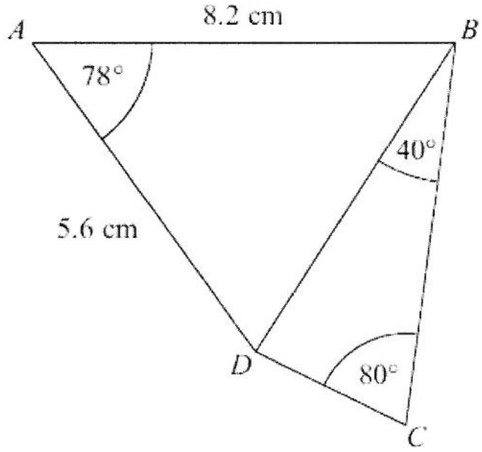
area $= \frac{1}{2}(8.7)(7.305\ldots)\sin 67$

$=$

29.3 (3 sf) cm²

(Total for Question is 5 marks)

3. *ABCD* is a quadrilateral.
Diagram **NOT** accurately drawn
Work out the length of *DC*.
Give your answer correct to 3 significant figures.



$BD = \sqrt{8.2^2 + 5.6^2 - 2(8.2)5.6\cos 78}$

$= 8.916579519$

$\boxed{ANS}$!

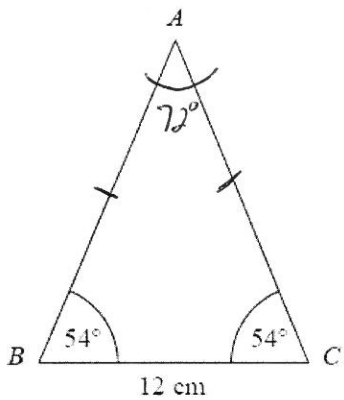
$\frac{DC}{\sin 40} = \frac{8.91\ldots}{\sin 80}$

$DC = \frac{8.91\ldots \times \sin 40}{\sin 80} =$

5.82 (3 sf) .... cm

(Total for Question is 6 marks)

4. Diagram **NOT** accurately drawn
ABC is an isosceles triangle.
Work out the area of the triangle.
Give your answer correct to 3 significant figures.



$\frac{12}{\sin 72} = \frac{AC}{\sin 54}$

$AC = \frac{12 \sin 54}{\sin 72}$

$= 10.2078097$

$\boxed{ANS}$!

area $= \frac{1}{2}(10.20\ldots)12\sin 54$

$= 49.5$ (3 sf)

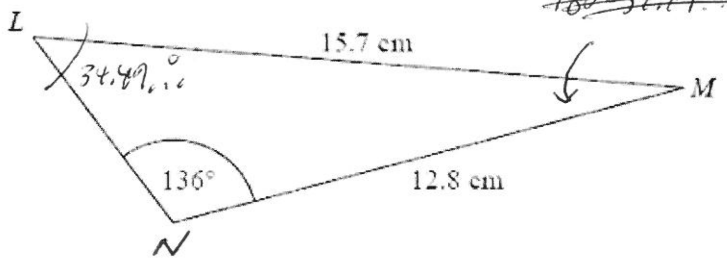
49.5 (3 sf) .... cm^2

(Total for Question is 4 marks)

5. Diagram **NOT** accurately drawn

$180 - 136 - 34.49\ldots$

~~180 − 34.49…~~



The diagram shows triangle *LMN*.
Calculate the length of *LN*.
Give your answer correct to 3 significant figures.

$$\frac{\sin 136}{15.7} = \frac{\sin N\hat{L}M}{12.8} \qquad \frac{LN}{\sin 9.5\ldots} = \frac{15.7}{\sin 136}$$

$$N\hat{L}M = \sin^{-1}\left(\frac{12.8 \sin 136}{15.7}\right) \qquad LN = \frac{15.7 \sin 9.5\ldots}{\sin 136}$$

$$= 34.49578985^\circ$$

$$N\hat{M}L = 180 - 34.49\ldots$$

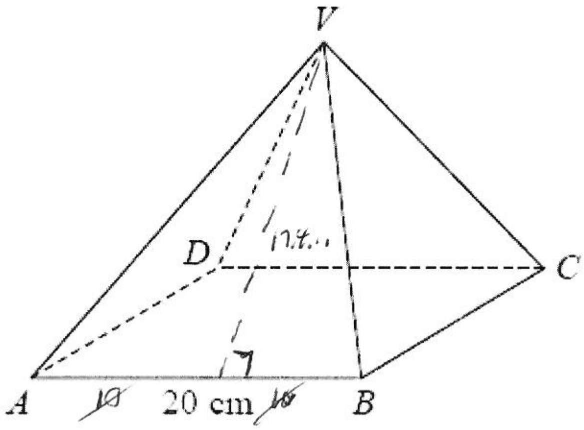
$$= 9.50421015$$

$\boxed{ANS}$

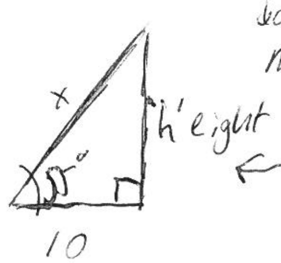
3.73 (3 sf) .... cm

(Total for Question is 5 marks)

6. *VABCD* is a solid pyramid.
ABCD is a square of side 20 cm.
The angle between any sloping edge and the plane *ABCD* is 55°
Calculate the surface area of the pyramid.
Give your answer correct to 2 significant figures.



'side view'



actually don't need this

triangular faces: $4 \times \frac{1}{2}(20)(17.4\ldots) = 697.3787\ldots$

base: $20^2 = 400$

$1097.378\ldots$

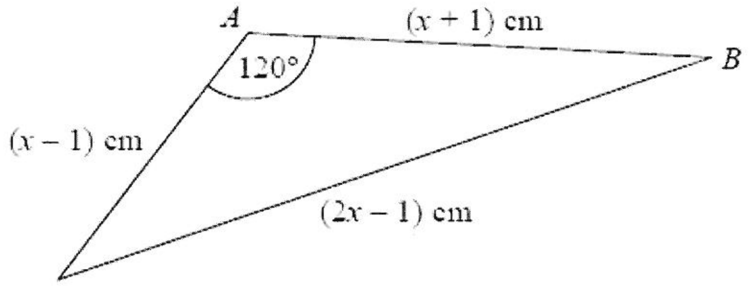
$$\cos 55^\circ = \frac{10}{x}$$

$$x = \frac{10}{\cos 55} = 17.43446796$$

1 100 (2 sf) ....cm^2

(Total for question = 5 marks)

7. The diagram shows triangle *ABC*.
The area of triangle *ABC* is $k\sqrt{3}$ cm^2.



Find the exact value of *k*.

$$\text{area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \underbrace{(x+1)(x-1)}_{\text{diff 2 squares}} \sin 120^\circ$$

$$= \frac{1}{2}(x^2-1)\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4}(x^2-1)$$

$$= \frac{\sqrt{3}}{4}x^2 - \frac{\sqrt{3}}{4}$$

$$\frac{\sqrt{3}}{4}(x^2-1) = k\sqrt{3}$$

$$\frac{\frac{\sqrt{3}}{4}(x^2-1)}{\sqrt{3}} = k$$

$$\frac{\frac{\sqrt{3}}{4}}{\sqrt{3}} = \frac{1}{4} \qquad \frac{1}{4}(x^2-1) = k$$

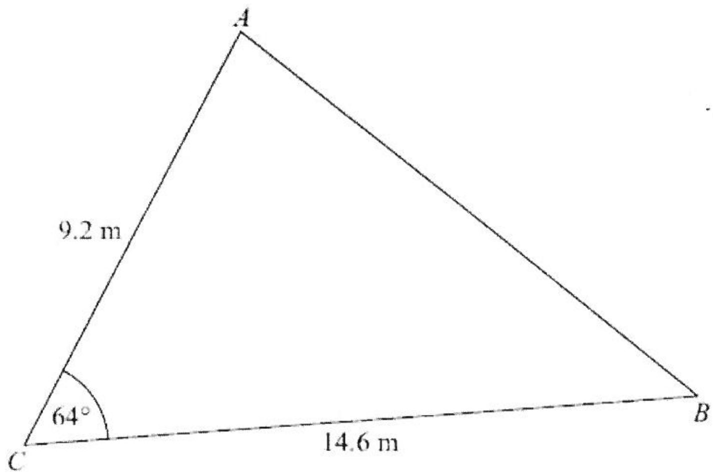
k = ...$\frac{1}{4}(x^2-1)$......

(Total for question = 7 marks)

8. Diagram **NOT** accurately drawn

AC = 9.2 m
BC = 14.6 m
Angle *ACB* = 64°

(a) Calculate the area of the triangle *ABC*.
Give your answer correct to 3 significant figures.



$$\text{area} = \frac{1}{2}(9.2)14.6 \sin 64$$

$$= 60.4 \text{ m}^2 \ (3\text{sf})$$

......60.4 (3 sf)...... m²
(2)

(b) Calculate the length of *AB*.
Give your answer correct to 3 significant figures.

$$AB = \sqrt{9.2^2 + 14.6^2 - 2(9.2)14.6\cos 64}$$

......13.4 m (3 sf)......
(3)

(Total for Question is 5 marks)

Area under Graphs

Things to remember:

- Velocity is speed with direction
- Acceleration and deceleration is given by the gradient of the graph $\left(\frac{rise}{run}\right)$
- The distance travelled is given by the area under the graph.

Questions:

1. A car has an initial speed of u m/s.
   The car accelerates to a speed of $2u$ m/s in 12 seconds.
   The car then travels at a constant speed of $2u$ m/s for 10 seconds.
   Assuming that the acceleration is constant, show that the total distance, in metres, travelled by the car is $38u$.

initial speed = u m/s

terminal speed = 2u m/s

constant acceleration means average speed $= \frac{u + 2u}{2} = \frac{3}{2}u$

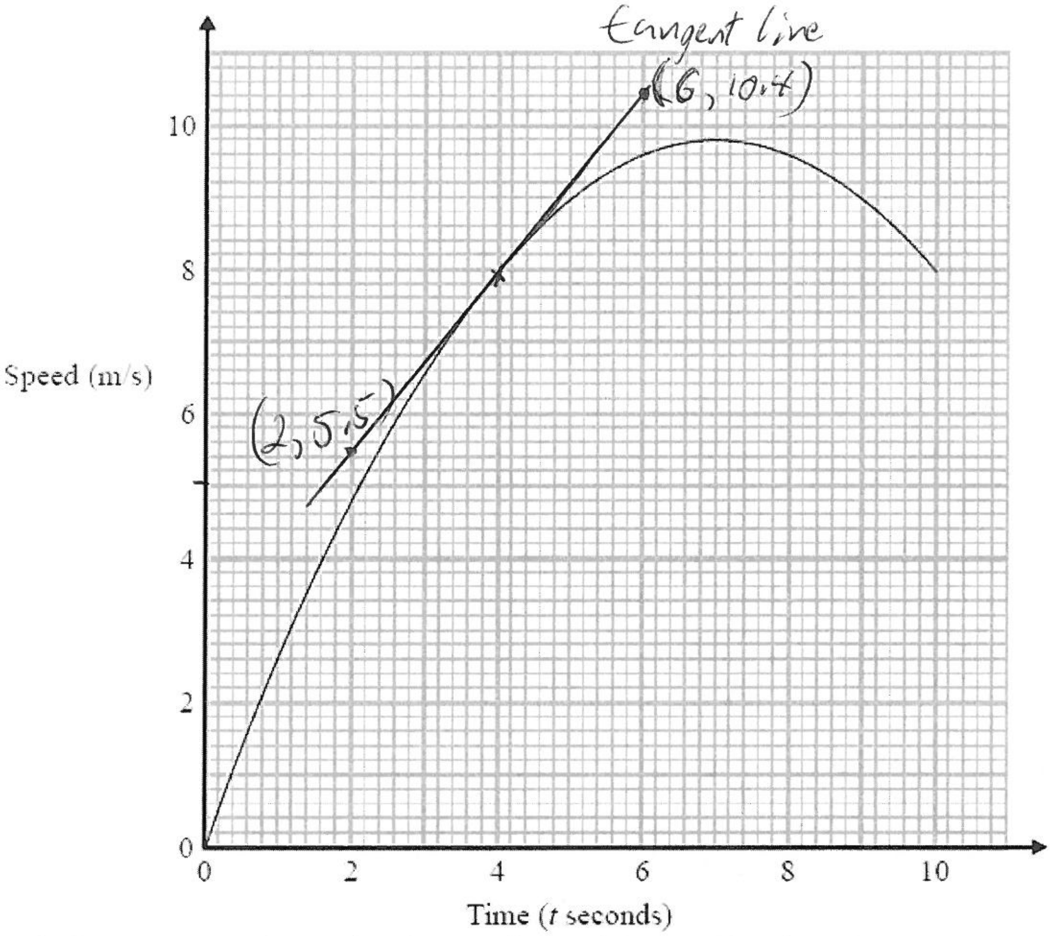
$\frac{3}{2}u$ m/s × 12 seconds = 18m

$2u$ m/s × 10 sec = 20 m

18 + 20 = 38

(Total for question = 4 marks)

2. Karol runs in a race.
The graph shows her speed, in metres per second, t seconds after the start of the race.



(a) Calculate an estimate for the gradient of the graph when $t = 4$
You must show how you get your answer.

$m = \frac{\Delta y}{\Delta x} = \frac{10.4 - 5.5}{6 - 2.0} = \frac{4.9}{4} = 1.225$

1.225

(3)

(b) Describe fully what your answer to part (a) represents.

change in y / change in x

change in speed / change in time = acceleration

(2)

(c) Explain why your answer to part (a) is only an estimate.

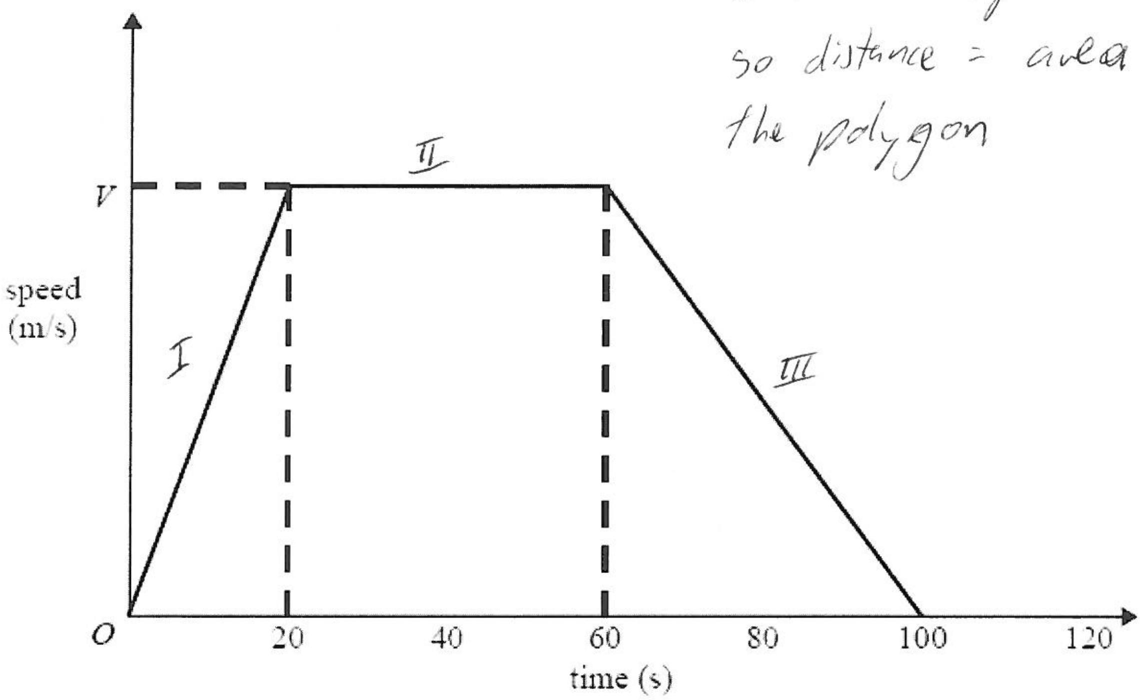
I have attempted to draw an accurate tangent line but the precision of the measurement is limited by my ability to draw accurately, the magnification of the drawing, etc.

(1)

(Total for question = 6 marks)

3. Here is a speed-time graph for a car journey.
The journey took 100 seconds.

Distance = speed × time
so distance = area under the polygon



The car travelled 1.75km in the 100 seconds.

(a) Work out the value of *V*.

$1750 = \frac{1}{2} \times 20 \times V + 40 \times V + \frac{1}{2} \times 40 \times V$ $\qquad 1750 \div 70 = 25$

meters

$= 10V + 40V + 20V$

$= 70V$

.......25 m/s.......

(3)

(b) Describe the acceleration of the car for each part of this journey.

~~Dr~~ acc = $\frac{\Delta s}{\Delta t}$

I : $\frac{25}{20} = 1\frac{1}{4}$ m/s^2 accelerating

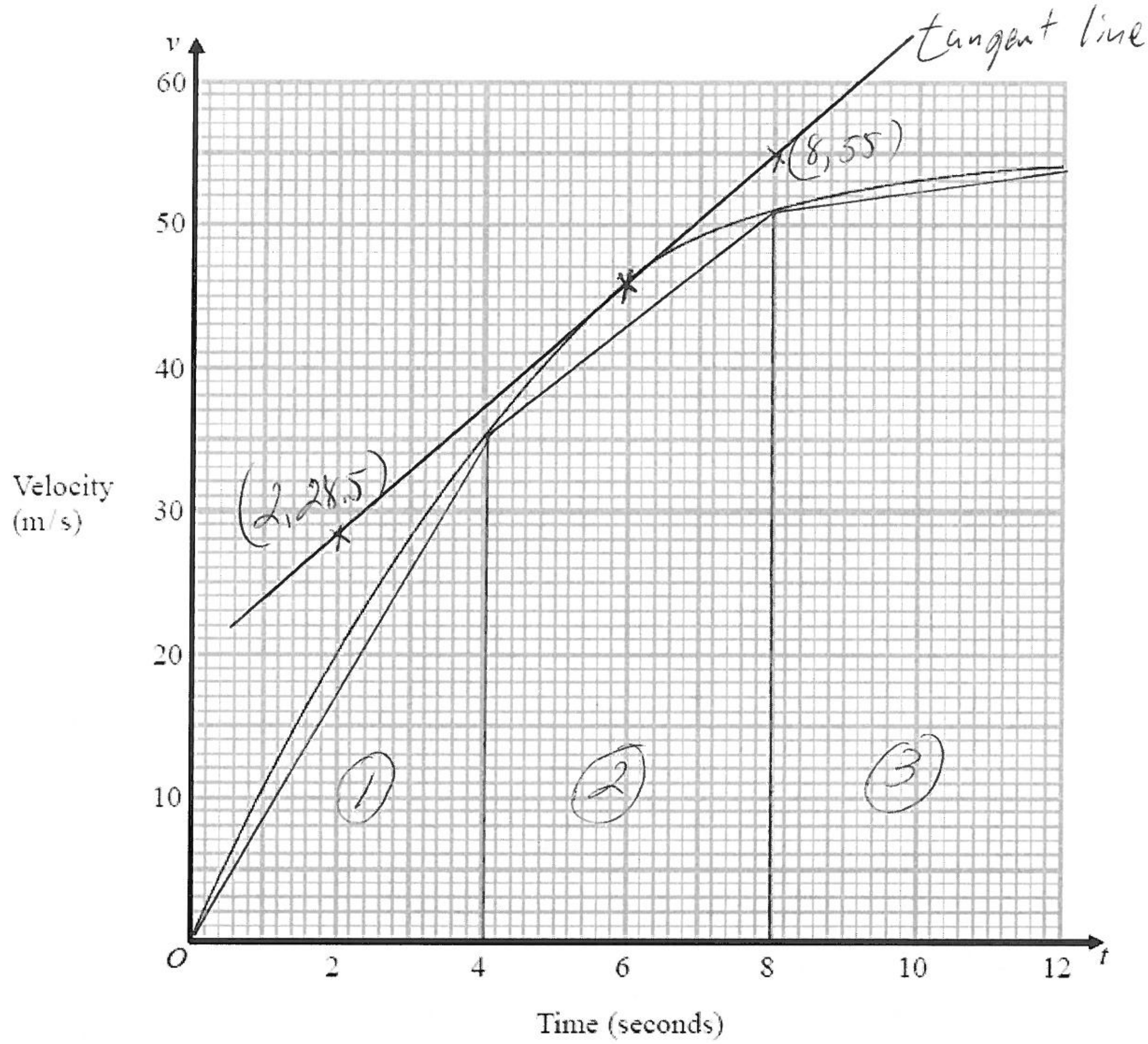
II : no acceleration, steady speed

III : $\frac{-25}{40} = -\frac{5}{8}$ ms^{-2} decelerating

(2)

(Total for question = 5 marks)

4. The graph shows information about the velocity, *v* m/s, of a parachutist *t* seconds after leaving a plane.



(a) Work out an estimate for the acceleration of the parachutist at $t = 6$

$\frac{55-28.5}{8-2} = 4.4 \, ms^{-2}$

4.4 m/s²
(2)

(b) Work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane.
Use 3 strips of equal width.

1: $\frac{1}{2} \times 4 \times 35 = 70$

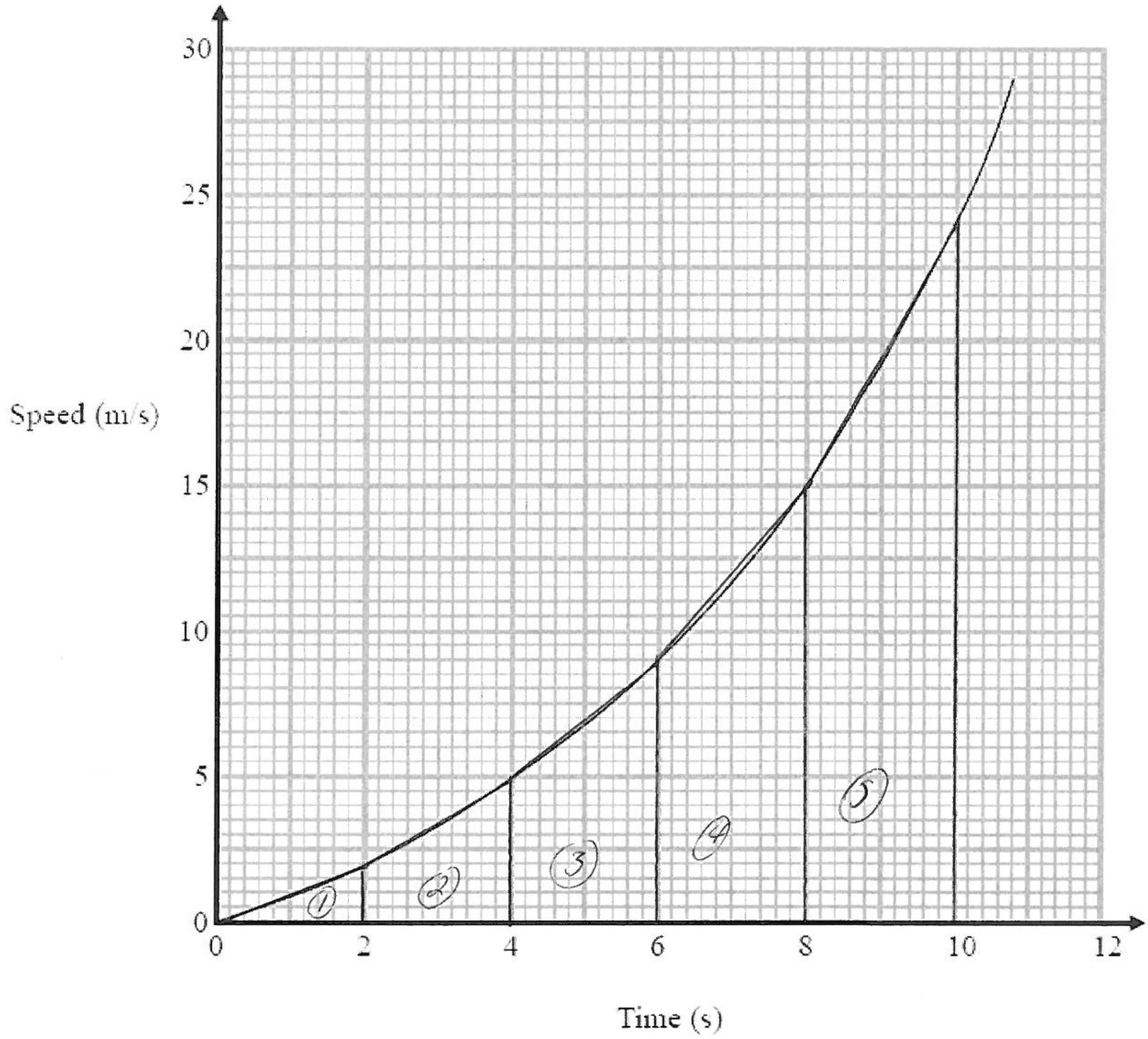
2: $\frac{35+51}{2} \times 4 = 172$

3: $\frac{51+54}{2} \times 4 = 210$

$70 + 172 + 210 = 452$

452 m
(3)
(Total for question is 5 marks)

5. Here is a speed-time graph for a car.



(a) Work out an estimate for the distance the car travelled in the first 10 seconds.
Use 5 strips of equal width.

1: $\frac{1}{2}(2)2 = 2$

2: $\frac{2+5}{2}(2) = 7$

3: $\frac{9+5}{2}(2) = 14$

4: $\frac{15+9}{2}(2) = 24$

5: $\frac{24+15}{2}(2) = 49$

$2 + 7 + 14 + 24 + 49 = 96$

........96........ m

(3)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance?
Give a reason for your answer.

Overestimate. Each triangle / trapezium covers slightly more area than that under the curve

(1)

(Total for question = 4 marks)

Histograms

Things to remember:

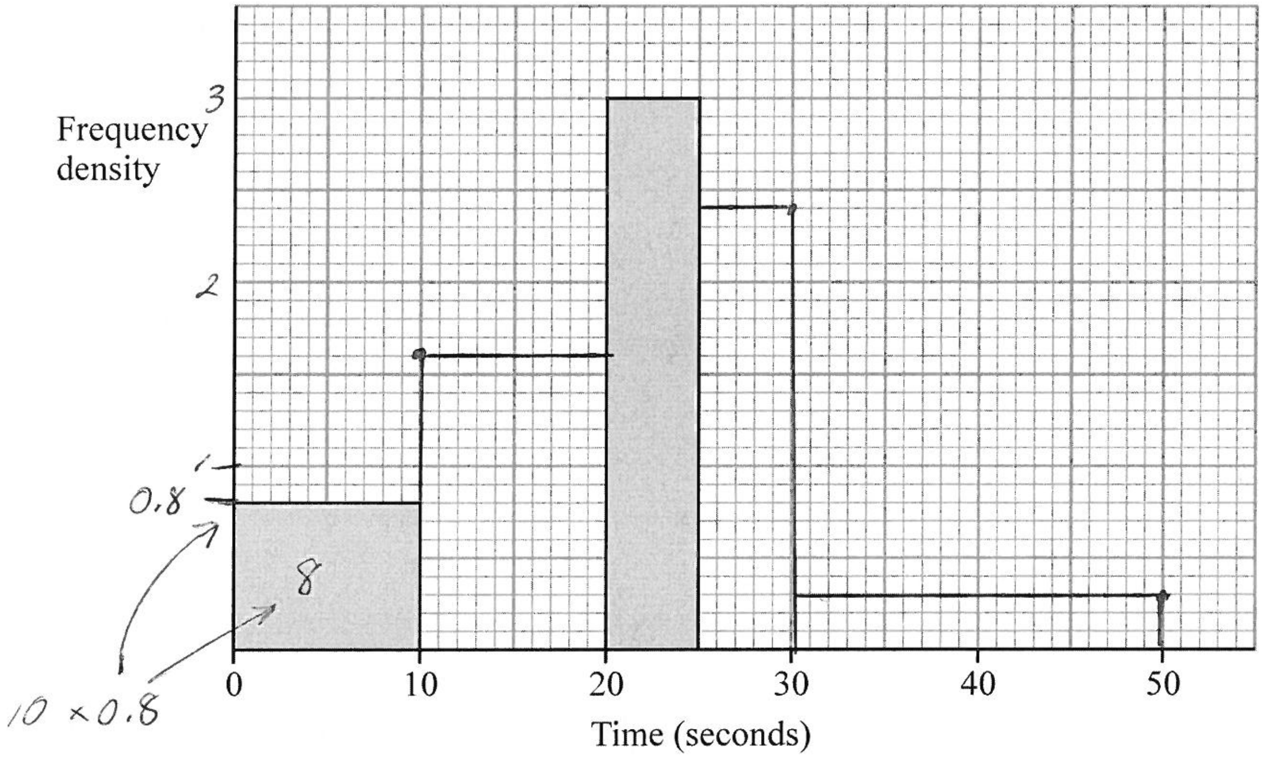
- Frequency = Frequency Density x Class Width;
- The y-axis will always be labelled “frequency density”;
- The x-axis will have a continuous scale.

Questions:

1. One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table. She used this information to complete the frequency table.

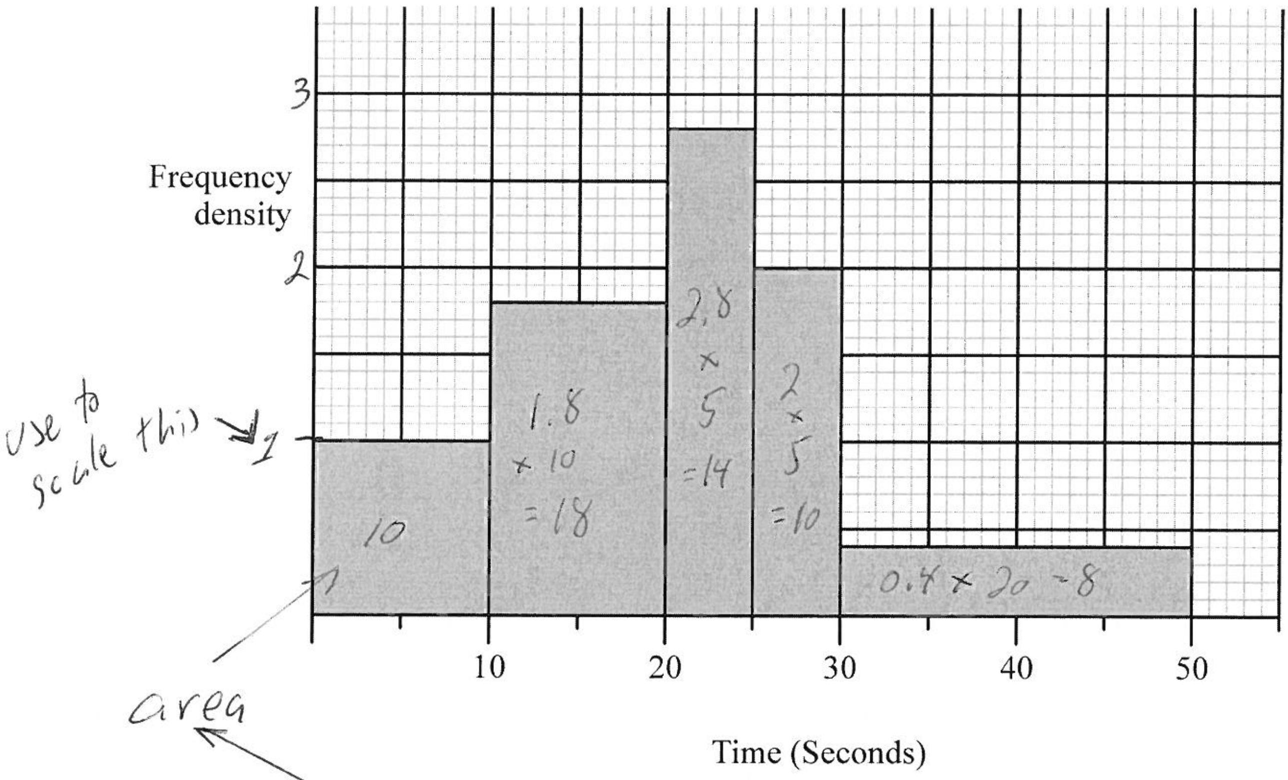
Time (t seconds)	Frequency	class width	fd(= F ÷ CW)
$0 < t \leq 10$	8	10	0.8
$10 < t \leq 20$	16	10	1.6
$20 < t \leq 25$	15	5	3
$25 < t \leq 30$	12	5	2.4
$30 < t \leq 50$	6	20	0.3

(a) Use the table to complete the histogram.



(3)

On Tuesday she conducted a similar survey and drew the following histogram from her results.

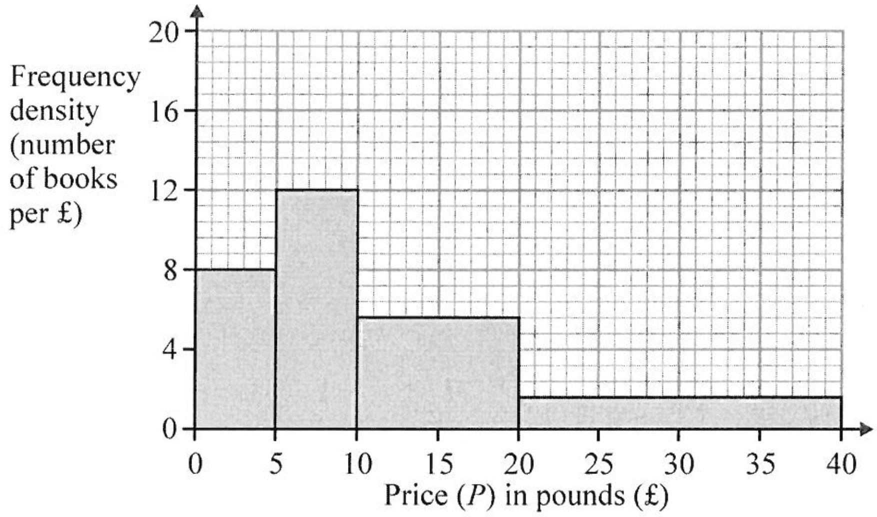


(b) Use the histogram for Tuesday to complete the table.

Time (t seconds)	Frequency
$0 < t \leq 10$	10
$10 < t \leq 20$	18
$20 < t \leq 25$	14
$25 < t \leq 30$	10
$30 < t \leq 50$	8

(2)
(Total 5 marks)

2. This histogram gives information about the books sold in a bookshop one Saturday.



(a) Use the histogram to complete the table.

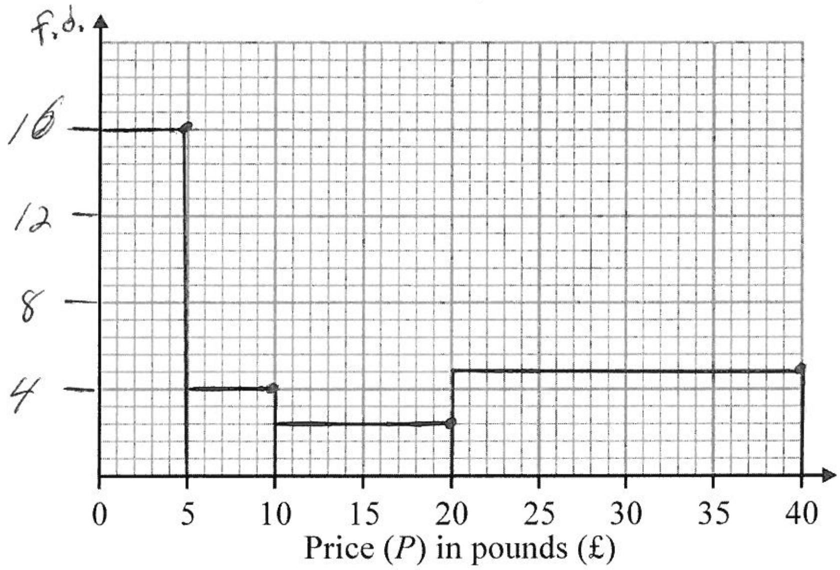
Price (P) in pounds (£)	Frequency
$0 < P \le 5$	$8 \times 5 = 40$
$5 < P \le 10$	$12 \times 5 = 60$
$10 < P \le 20$	$5.6 \times 10 = 56$
$20 < P \le 40$	$1.6 \times 20 = 32$

(2)

The frequency table below gives information about the books sold in a second bookshop on the same Saturday.

Price (P) in pounds (£)	Frequency	cw	f.d.
$0 < P \le 5$	80	5	16
$5 < P \le 10$	20	5	4
$10 < P \le 20$	24	10	2.4
$20 < P \le 40$	96	20	4.8

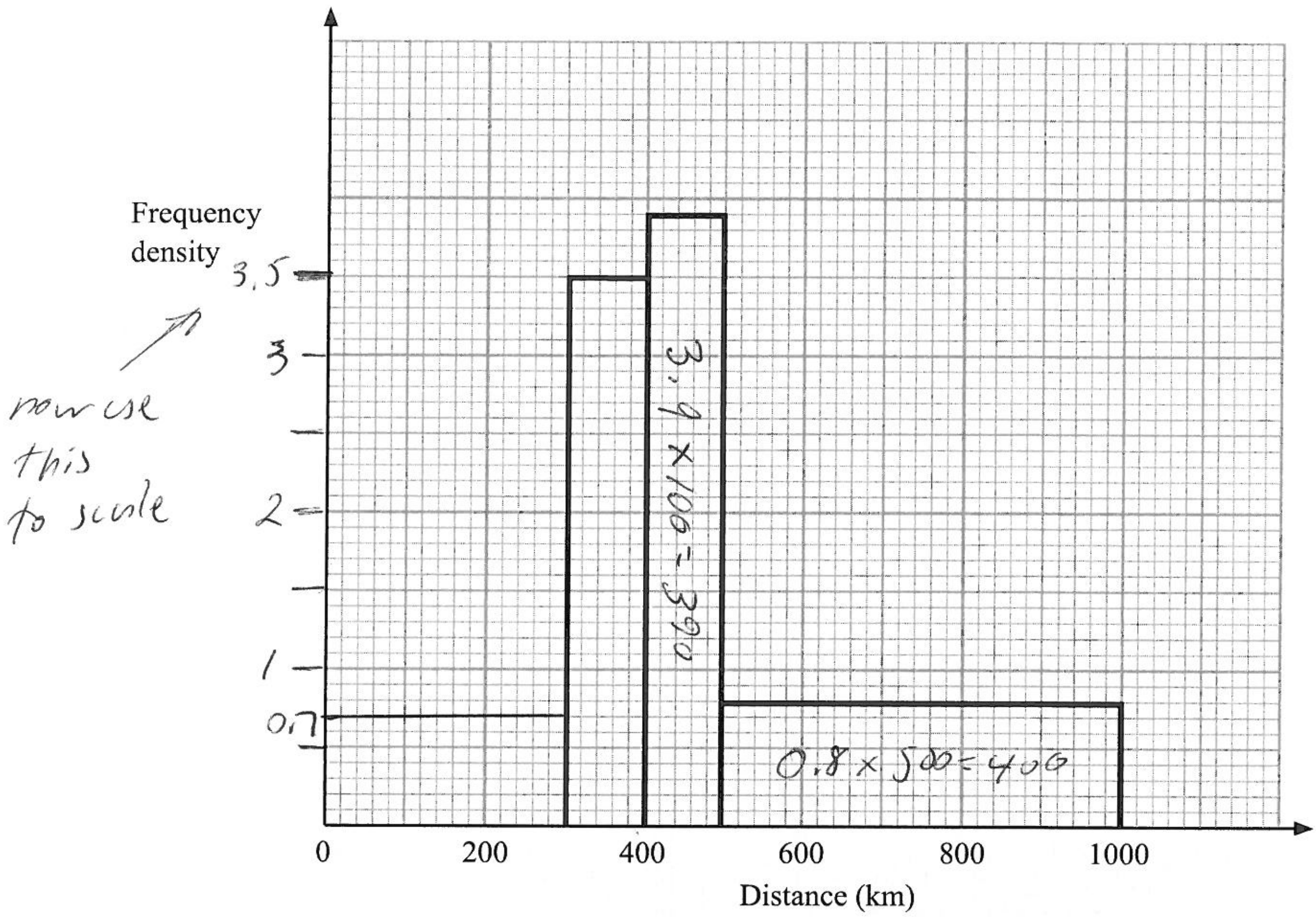
(b) On the grid below, draw a histogram to represent the information about the books sold in the second bookshop.



do this work to determine f.d. scale

(3)
(Total 5 marks)

3. The incomplete table and histogram give some information about the distances walked by some students in a school in one year.



(a) Use the information in the histogram to complete the frequency table.

Distance (d) in km	Frequency
$0 < d \leq 300$	210
$300 < d \leq 400$	350
$400 < d \leq 500$	390
$500 < d \leq 1000$	400

c.w. f.d.
300 0.7
100 3.5
100
500

(2)

(b) Use the information in the table to complete the histogram.

(1)

(Total 3 marks)

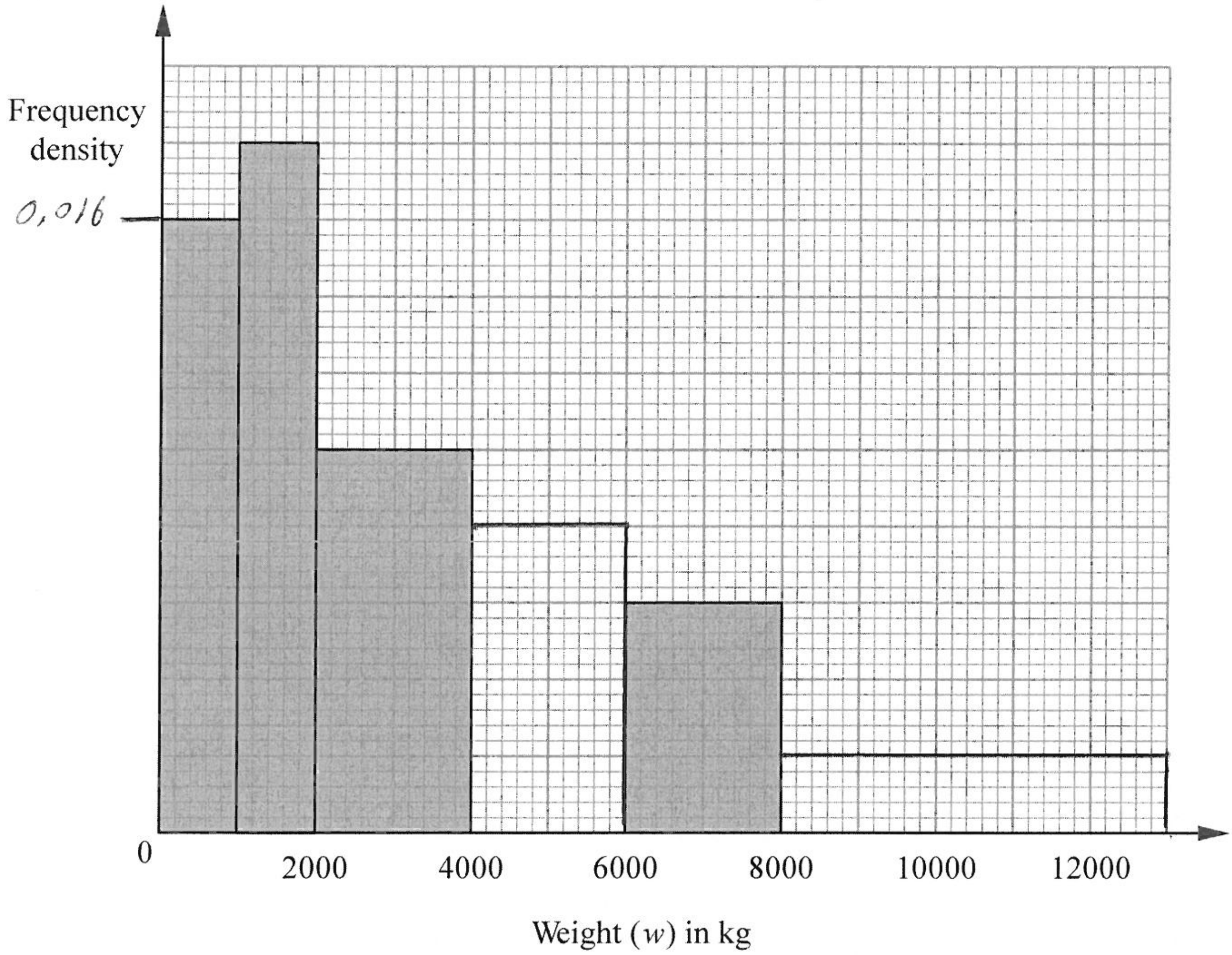
4. The incomplete histogram and table show information about the weights of some containers.

Weight (w) in kg	**Frequency**	class width ~~(000)~~	freq. density (F ÷ CW)
$0 < w \leq 1000$	16	1000	0.016
$1000 < w \leq 2000$	18	1000	0.018
$2000 < w \leq 4000$	20	2000	0.01
$4000 < w \leq 6000$	16	2000	0.008
$6000 < w \leq 8000$	12	2000	0.006
$8000 < w \leq 12000$	8	4000	0.002

(a) Use the information in the histogram to complete the table.

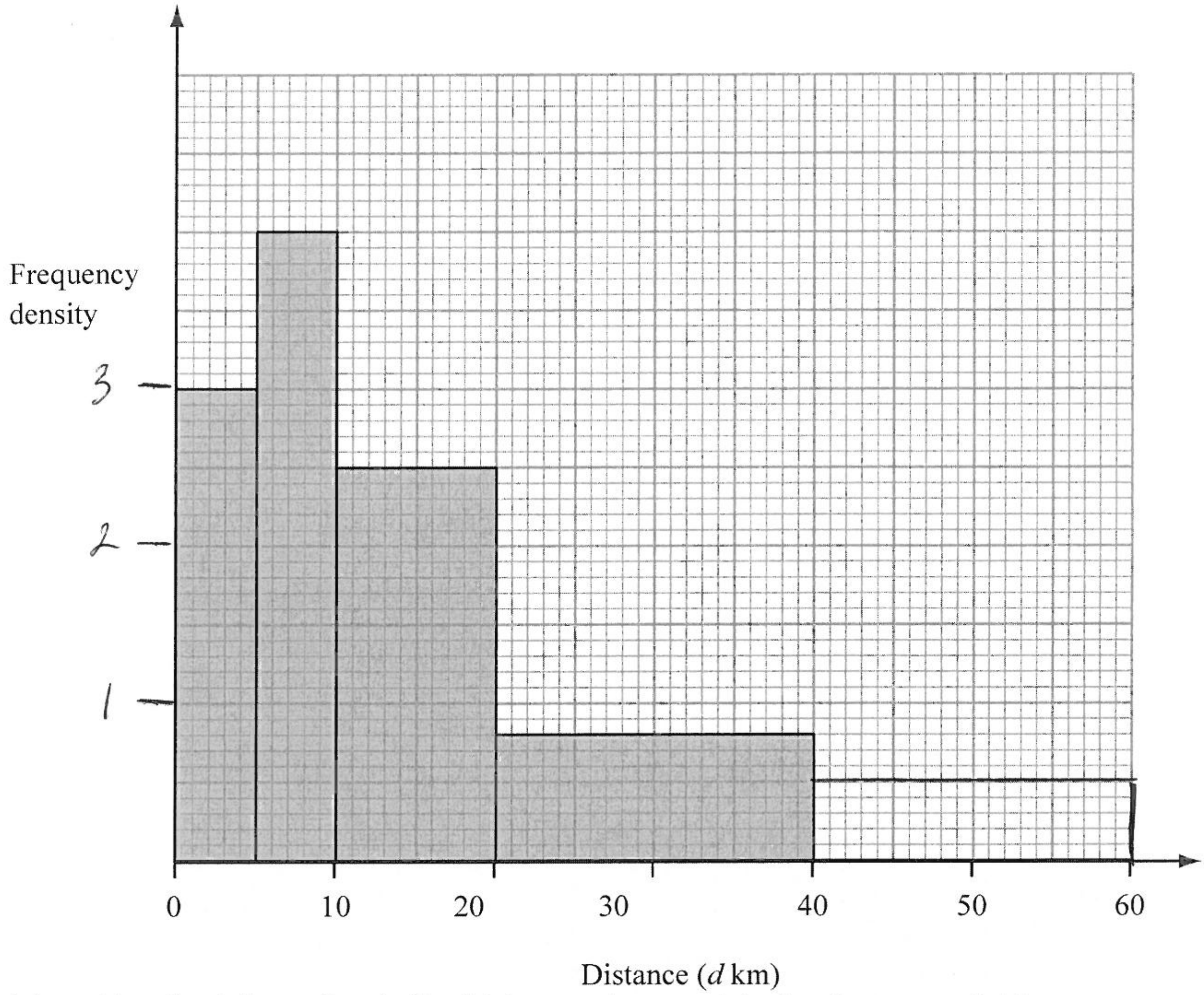
(2)

(b) Use the information in the table to complete the histogram.



(2)
(Total 4 marks)

5. The incomplete histogram and table give some information about the distances some teachers travel to school.



(a) Use the information in the histogram to complete the frequency table.

Distance (d km)	Frequency	c.w.	f.d.
$0 < d \leq 5$	15	5	3
$5 < d \leq 10$	20	5	4
$10 < d \leq 20$	25	10	2.5
$20 < d \leq 40$	16	20	0.8
$40 < d \leq 60$	10	20	0.5

(2)

(b) Use the information in the table to complete the histogram.

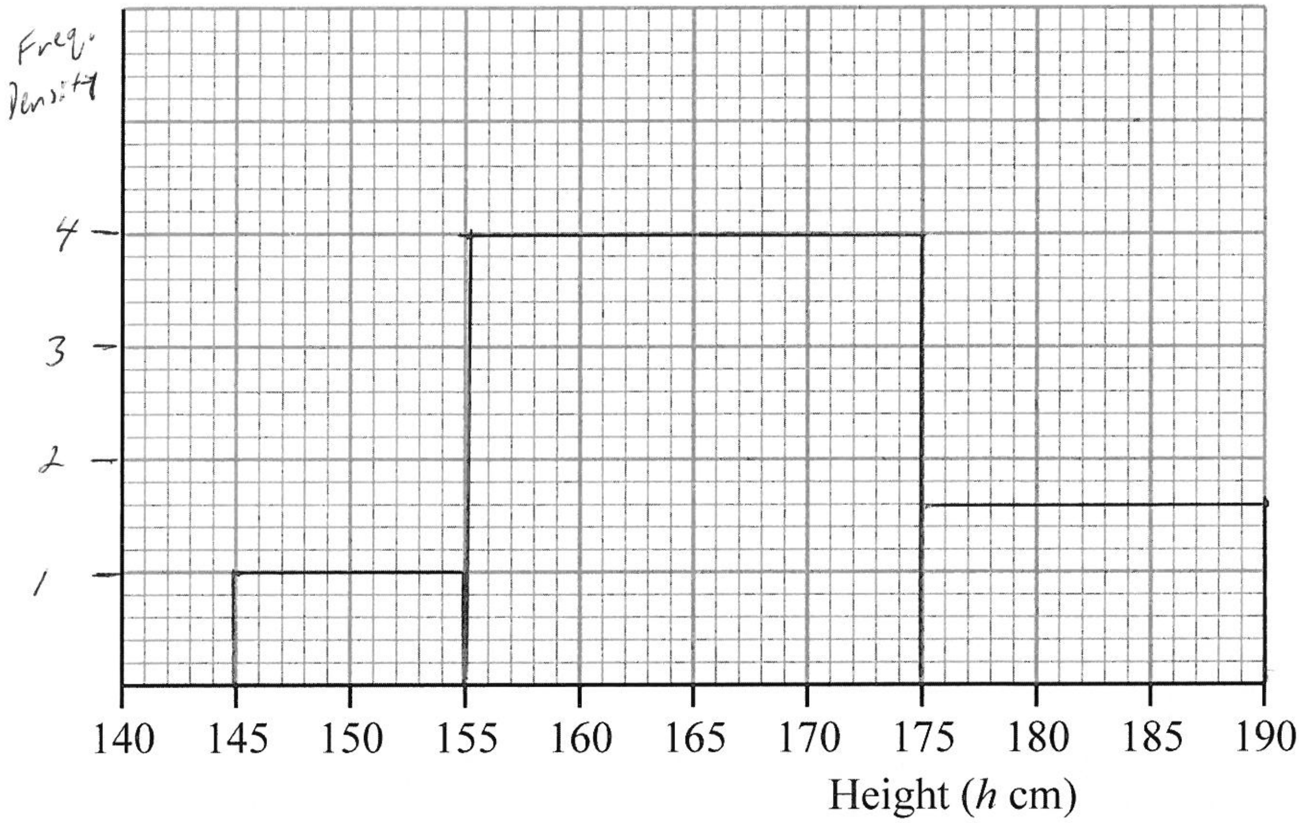
(1)

(Total 3 marks)

6. The table gives information about the heights, in centimetres, of some 15 year old students.

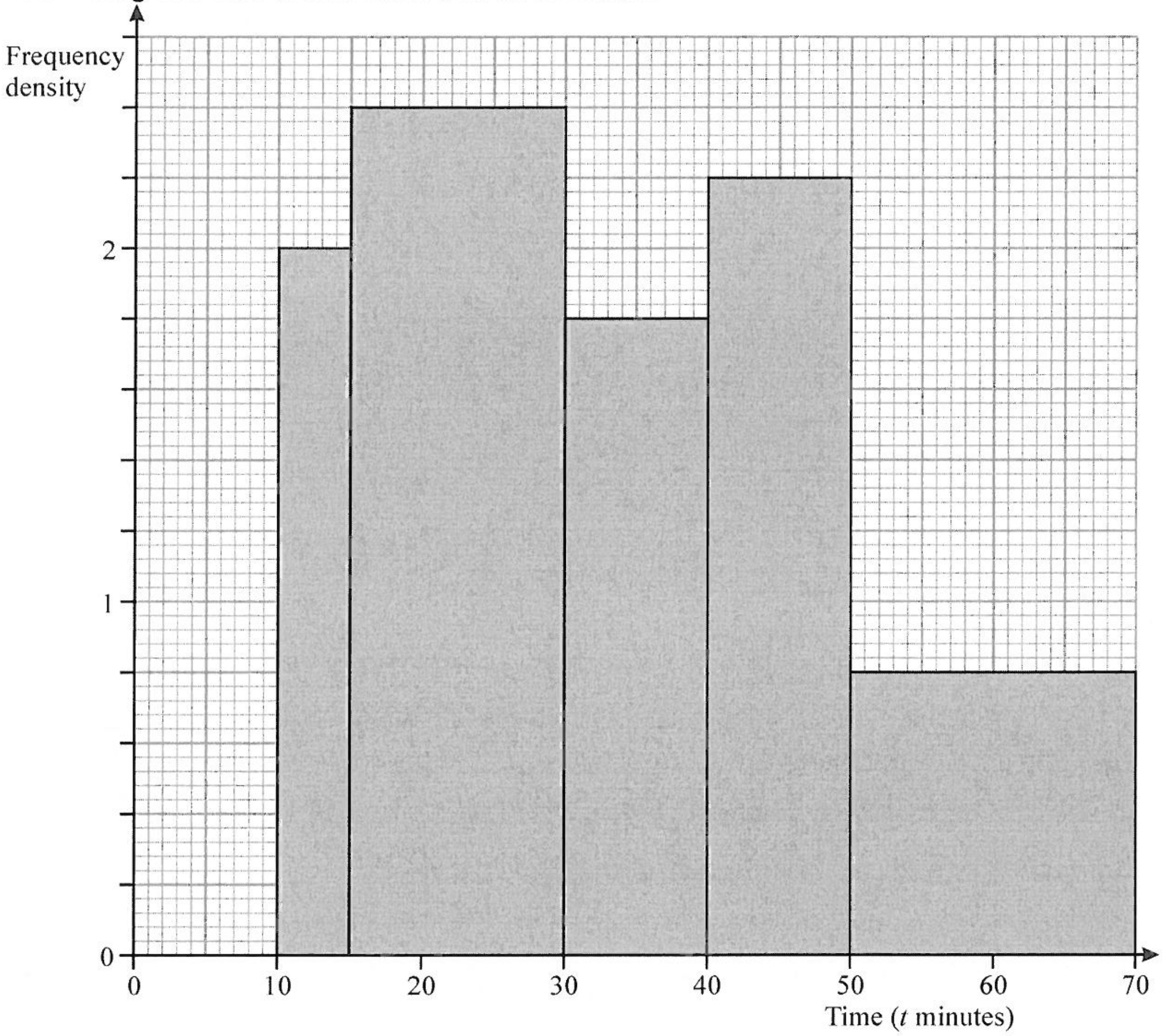
Class width	10	20	15
Height (h cm)	$145 < h \leq 155$	$155 < h \leq 175$	$175 < h \leq 190$
Frequency	10	80	24
F.d.	1	4	8/5 (1.6)

Use the table to draw a histogram.



(Total 3 marks)

7. A teacher asked some year 10 students how long they spent doing homework each night. The histogram was drawn from this information.



Use the histogram to complete the table.

Time (*t* minutes)	Frequency	Class Width	Freq. Density
10 ≤ *t* < 15	10	5	2
15 ≤ *t* < 30	36	15	2.4
30 ≤ *t* < 40	18	10	1.8
40 ≤ *t* < 50	22	10	2.2
50 ≤ *t* < 70	16	20	0.8

(Total 2 marks)

Moving Averages

Things to remember:

- In this context, averages means the mean (add the numbers and divide by how many there were.
- Moving averages are used to identify trends in data – peaks, troughs, increasing and decreasing trends.

Questions:

1. The table shows the number of computer games sold in a supermarket each month from January to June.

Jan	Feb	Mar	Apr	May	Jun
147	161	238	135	167	250

Work out the three month moving averages for this information.

Mar $= \frac{147+161+238}{3} = 182$

Jun $= \frac{135+167+250}{3} = 184$

Apr $= \frac{161+238+135}{3} = 178$

May $= \frac{238+135+167}{3} = 180$

182 178 180 184

(Total 2 marks)

2. The table shows the number of digital cameras Bytes sold each month in the first six months of 2005.

Month	January	February	March	April	May	June
Number of digital cameras sold	30	19	20	15	27	39

The first 3-month moving average for this data is 23

Work out the **second** 3-month moving average for this data.

Apr: $\frac{19+20+15}{3} = 18$

NB: There are 4 moving averages calculable from 6 months' data
Apr is the second

18

(Total 2 marks)

3. The table shows the number of orders received each month by a small company.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Number of orders received	23	31	15	11	19	16	20	13

Work out the first two 4-month moving averages for this data.

Apr: $\frac{23+31+15+11}{4} = 20$

May: $\frac{31+15+11+19}{4} = 19$

20 and 19

(Total 3 marks)

4. A shop sells DVD players.
The table shows the number of DVD players sold in every three-month period from January 2003 to June 2004.

Year	Months	Number of DVD players sold
2003	Jan – Mar	58
	Apr – Jun	64
	Jul – Sep	86
	Oct – Dec	104
2004	Jan – Mar	65
	Apr – Jun	70

Cum. total — MA = CT/4

312 — 78
319 — 79 3/4
325 — 81 1/4

(a) Calculate the set of four-point moving averages for this data.

78, 79 3/4, 81 1/4

(2)

(b) What do your moving averages in part (a) tell you about the trend in the sale of DVD players?

The sales trend is increasing

(1)

(Total 3 marks)

5. Paul and Carol open a new shop in the High Street.
The table shows the monthly takings in each of the first four months.

Month	Jan	Feb	March	April
Monthly takings (£)	9375	8907	9255	9420

Work out the 3-point moving averages for this information.

Cum total — 27507 27582

MA = CT ÷ 3 — 9169 9194

9169 9194

(Total 2 marks)

6. The owner of a music shop recorded the number of CDs sold every 3 months.
The table shows his records from January 2004 to June 2005.

Year	Months	Number of CDs	Cumulative total	MA = CT/4
2004	Jan – Mar	270		
	Apr – Jun	324		
	Jul – Sept	300		
	Oct – Dec	258	1152	288
2005	Jan – Mar	309	1191	297 3/4
	Apr – Jun	335	1202	300 1/2

(a) Calculate the complete set of four-point moving averages for this information.

288 297 3/4 300 1/2

(2)

(b) What trend do these moving averages suggest?

The moving average data suggests an increasing trend in sales

(1)
(Total 3 marks)

7. The table shows some information about student absences.

Term	Autumn 2003	Spring 2004	Summer 2004	Autumn 2004	Spring 2005	Summer 2005
Number of absences	408	543	351	435	582	372

Work out the three-point moving averages for this information. The first two have been done for you.

Spring 2005: $\frac{351+435+582}{3} = 456$

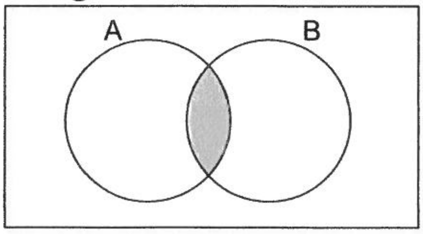
Summer 2005: $\frac{435+582+372}{3} = 463$

434, 443, 456, 463

(Total 2 marks)

Set Theory

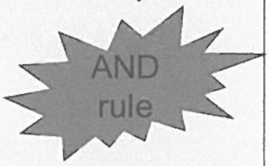
Things to remember:

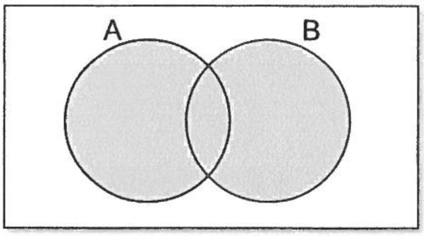


The **intersection** is where two sets overlap.

$$A \cap B$$

This means **A and B**.

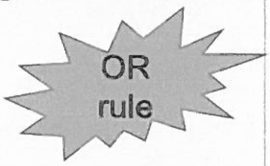


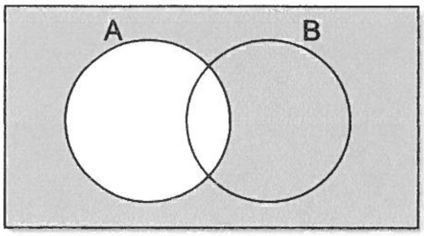


If you put two sets together, you get the **union**.

$$A \cup B$$

This means **A or B**.





The **complement of A** is the region that is not A.

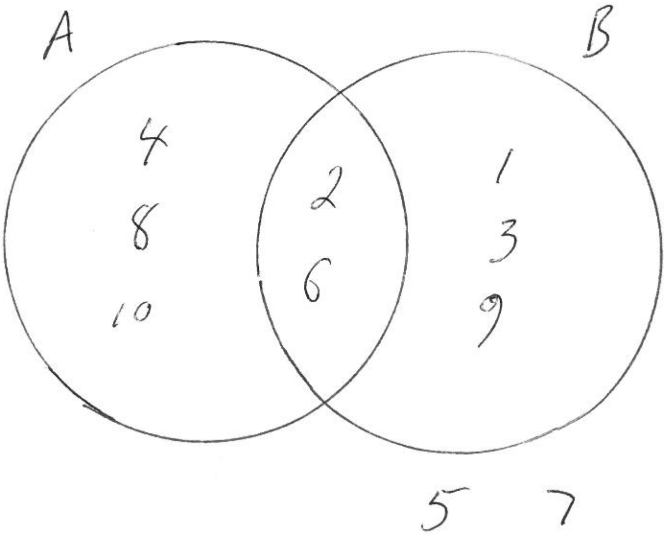
$$A'$$

This means **not A**.

Questions:

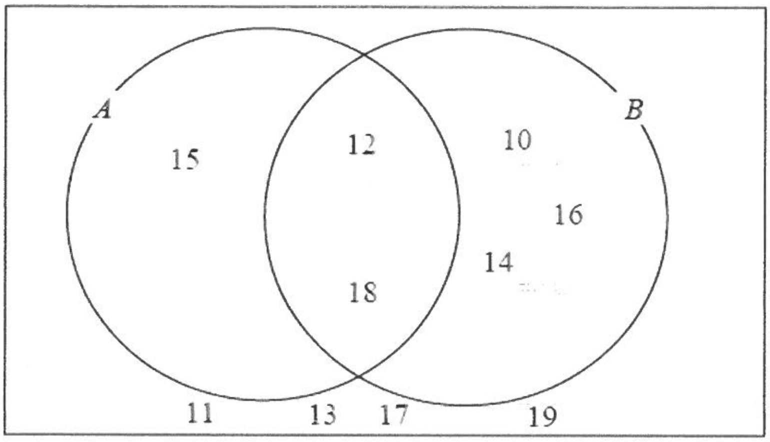
1.

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{multiples of } 2\}$
$A \cap B = \{2, 6\}$
$A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10\}$
Draw a Venn diagram for this information.



(Total for question is 4 marks)

2. Here is a Venn diagram.



(a) Write down the numbers that are in set

(i) $A \cup B$

15, 12, 18, 10, 16, 14

(ii) $A \cap B$

12, 18

(2)

One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set A'

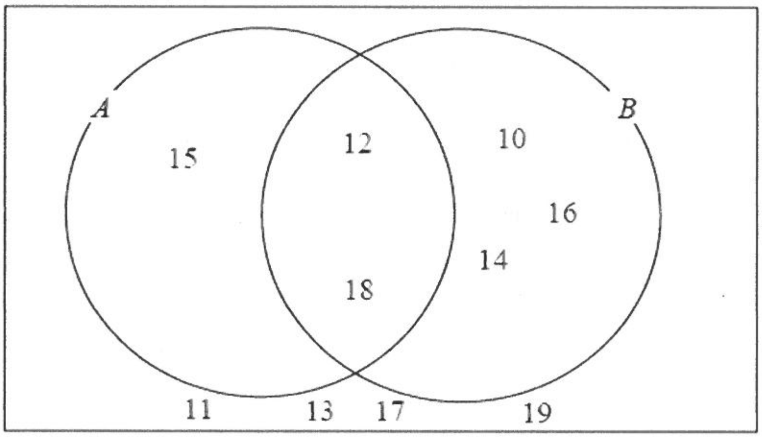
$\frac{7}{10}$

....

(2)

(Total for question = 4 marks)

3. Here is a Venn diagram.



(a) Write down the numbers that are in set

(i) $A \cup B$

....

(ii) $A \cap B$

....

(2)

One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set A'

See above

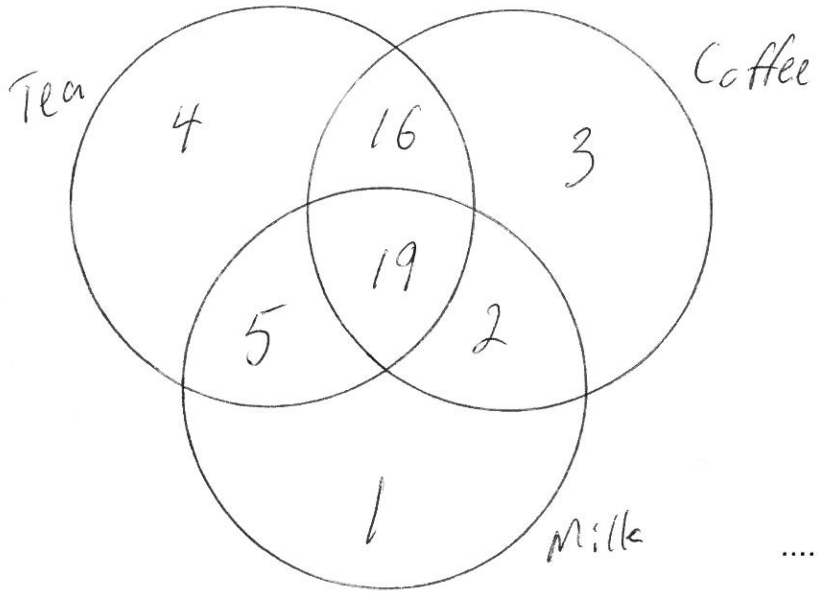
....

(2)

(Total for question = 4 marks)

4. Sami asked 50 people which drinks they liked from tea, coffee and milk.
All 50 people like at least one of the drinks
19 people like all three drinks.
16 people like tea and coffee but do not like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.
Sami selects at random one of the 50 people.

(a) Work out the probability that this person likes tea.



$44/50$

(4)

(b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

$$\frac{16+5}{44} = \frac{21}{44}$$

$21/44$

(2)

(Total for question = 6 marks)

Proportion

Things to remember:

- Start by checking the question for squares, cubes and roots;
- "x is directly proportional to y" looks like **x α y** or **x = ky**
- "x is inversely proportional to y" looks like **x α** $\frac{1}{y}$ or **x =** $\frac{k}{y}$

Questions:

1. The shutter speed, *S*, of a camera varies inversely as the square of the aperture setting, *f*.
When *f* = 8, *S* = 125

(a) Find a formula for *S* in terms of *f*.

$S \propto \frac{1}{f^2}$ $\qquad 125 = \frac{k}{8^2} = \frac{k}{64}$

$S = \frac{k}{f^2}$ $\qquad k = 125 \times 64$

$= 8000$

$S = \frac{8000}{f^2}$ ....

(3)

(b) Hence, or otherwise, calculate the value of *S* when *f* = 4

$S = \frac{8000}{f^2}$

$S = \frac{8000}{4^2} = \frac{8000}{16} = 500$

S = 500 ....

(1)

(Total 4 marks)

2. In a factory, chemical reactions are carried out in spherical containers.
The time, *T* minutes, the chemical reaction takes is directly proportional to the square of the radius, *R* cm, of the spherical container.
When *R* = 120, *T* = 32
Find the value of *T* when *R* = 150

$T \propto R^2$

$T = kR^2$

$32 = k(120)^2$

$k = \frac{32}{120^2} = \frac{1}{450}$

$T = \frac{1}{450} R^2$

$T = \frac{1}{450}(150)^2$

$= 50$

T = 50 ....

(Total 4 marks)

3. d is directly proportional to the square of t.
$d = 80$ when $t = 4$

(a) Express d in terms of t.

$d \propto t^2$

$d = kt^2$

$80 = k\,4^2 = 16k$

$k = \frac{80}{16} = 5$

$d = 5t^2$

$d = 5t^2$

(3)

(b) Work out the value of d when $t = 7$

$d = 5t^2$

$d = 5(7)^2$

$= 245$

$d =$ 245

(1)

(c) Work out the positive value of t when $d = 45$

$d = 5t^2$

$45 = 5t^2$

$\frac{45}{5} = 9 = t^2$

~~$t = \pm 9$~~ $t = \pm 3$

oops!

$t =$ ~~9~~ 3

(2)

(Total 6 marks)

4. The distance, D, travelled by a particle is directly proportional to the square of the time, t, taken. When $t = 40$, $D = 30$

(a) Find a formula for D in terms of t.

$D \propto t^2$

$D = kt^2$

$30 = k\,40^2$

$30 = 1600k$

$\frac{30}{1600} = \frac{3}{160} = k$

$D = \frac{3}{160}t^2$

$D = \frac{3}{160}t^2$

(3)

(b) Calculate the value of D when $t = 64$

$D = \frac{3}{160}t^2$

$D = \frac{3}{160}64^2$

$= 76\,4/5$

$76\,4/5$

(1)

(c) Calculate the value of t when $D = 12$
Give your answer correct to 3 significant figures.

$D = \frac{3}{160}t^2$

$12 = \frac{3}{160}t^2$

$640 = t^2$

$t = 8\sqrt{10}$

25.3 (3 s.f)

(2)

(Total 6 marks)

5. The time, T seconds, it takes a water heater to boil some water is directly proportional to the mass of water, m kg, in the water heater. When $m = 250$, $T = 600$

(a) Find T when $m = 400$

$T \propto m$

$T = km$

$600 = k(250)$

$k = \frac{600}{250} = 2\frac{2}{5}$

$T = 2\frac{2}{5}m$

$T = 2\frac{2}{5}(400)$

$= 960$

$T =$ ....960........

(3)

The time, T seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power, P watts, of the water heater.
When $P = 1400$, $T = 360$

(b) Find the value of T when $P = 900$

$T \propto \frac{1}{P}$

$T = \frac{k}{P}$

$360 = \frac{k}{1400}$

$k = 360 \times 1400 = 504000$

$T = \frac{504000}{P}$

$T = \frac{504000}{900} = 560$

$T =$ ....560........

(3)

(Total 6 marks)

6. A ball falls vertically after being dropped.
The ball falls a distance d metres in a time of t seconds.
d is directly proportional to the square of t.
The ball falls 20 metres in a time of 2 seconds.

(a) Find a formula for d in terms of t.

$d \propto t^2$

$d = kt^2$

$20 = k2^2$

$= 4k$

$\frac{20}{4} = k$

$5 = k$

$d = 5t^2$

$d =$ ....$5t^2$........

(3)

(b) Calculate the distance the ball falls in 3 seconds.

$d = 5t^2$

$d = 5(3)^2$

$= 5(9) = 45$

........45........ m

(1)

(c) Calculate the time the ball takes to fall 605 m.

$d = 5t^2$

$605 = 5t^2$

$\frac{605}{5} = t^2$

$121 = t^2$

$11 = t$

........11........ seconds

(3)

(Total 7 marks)

7. In a spring, the tension (T newtons) is directly proportional to its extension (x cm). When the tension is 150 newtons, the extension is 6 cm.

(a) Find a formula for T in terms of x.

$T \propto x$

$T = kx$

$150 = k(6)$

$k = \frac{150}{6} = 25$

$T = 25x$

$T = $ ......$25x$......

(3)

(b) Calculate the tension, in newtons, when the extension is 15 cm.

$T = 25x$

$T = 25(15)$

$= 375$

......375......newtons

(1)

(c) Calculate the extension, in cm, when the tension is 600 newtons.

$T = 25x$

$600 = 25x$

$\frac{600}{25} = x$

$x = 24$

......24......cm

(1)

(Total 5 marks)

8. f is inversely proportional to d.
When $d = 50$, $f = 256$
Find the value of f when $d = 80$

$f \propto \frac{1}{d}$

$f = \frac{k}{d}$

$256 = \frac{k}{50}$

$f = \frac{12800}{d}$

$f = \frac{12800}{80}$

$= 160$

$f = $ ......160......

(Total 3 marks)

$k = 256 \times 50$

$= 12800$

Percentages – reverse

Things to remember:

- Work out what the multiplier would have been;

$$\begin{tikzcd} \boxed{\text{Original amount}} \arrow[r, shift left, "\times \text{ multiplier}"] & \boxed{\text{New amount}} \arrow[l, shift left, "\div \text{ multiplier}"] \end{tikzcd}$$

Questions:

1. Loft insulation reduces annual heating costs by 20%.
After he insulated his loft, Curtley's annual heating cost was £520.
Work out Curtley's annual heating cost would have been, if he had not insulated his loft.

$520 = 0.8x$ NB: $0.8 = 100\% - 20\%$

$\frac{520}{0.8} = x = 650$

£ 650

(Total 3 marks)

2. In a sale, normal prices are reduced by 20%.

SALE - 20% OFF

Andrew bought a saddle for his horse in the sale.
The sale price of the saddle was £220.
Calculate the normal price of the saddle.

$220 = 0.8x$ NB: $0.8 = 100\% - 20\%$

$\frac{220}{0.8} = x = 275$

£ 275

(Total 3 marks)

3. Hajra's weekly pay this year is £240
This is 20% more than her weekly pay last year.
Bill says 'This means Hajra's weekly pay last year was £192'.
Bill is wrong,

(a) Explain why.

$240 = 1.2x$ $x = 200$ correct answer
Bill mistakenly subtracted 20% of 240

(1)

(b) Work out Hajra's weekly pay last year.

$240 = 1.2x$ NB: $1.2 = 100\% + 20\%$

$\frac{240}{1.2} = x = 200$

£ 200

(2)
(Total 3 marks)

4. The price of all rail season tickets to London increased by 4%.

(a) The price of a rail season ticket from Cambridge to London increased by £121.60
Work out the price before this increase.

$4\% \rightarrow 121.60$

$\times 25 \downarrow \qquad \downarrow \times 25$

$100\% \rightarrow 3040$

£ ....3040....

(2)

(b) After the increase, the price of a rail season ticket from Brighton to London was £2828.80
Work out the price before this increase.

$2828.80 = 1.04x$

$\frac{2828.80}{1.04} = x = 2720$

£ ....2720....

(3)

(Total 5 marks)

5. In a sale, normal prices are reduced by 25%.
The sale price of a saw is £12.75
Calculate the normal price of the saw.

$12.75 = 0.75x$

$\frac{12.75}{0.75} = x = 17$

£ ....17....

(Total 3 marks)

6. In a sale, normal prices are reduced by 12%.
The sale price of a DVD player is £242.
Work out the normal price of the DVD player.

$242 = 0.88x$

$\frac{242}{0.88} = x = 275$

£ ....275....

(Total 3 marks)

7. A garage sells cars.
It offers a discount of 20% off the normal price for cash.
Dave pays £5200 cash for a car.
Calculate the normal price of the car.

$5200 = 0.8x$

$\frac{5200}{0.8} = x = 6500$

£ ....6500....

(Total 3 marks)

Useful websites:

www.mathswatchvle.com
(Video explanations and questions)
Centre ID: twgash
Username: firstname
Password: lastname

www.methodmaths.com
(Past papers online that get instantly marked)
Centre ID: wga
Username: firstname
Password: lastname

www.hegartymaths.com
(Online tutorials and quizzes)
Login: first name and last name are case sensitive

www.bbc.co.uk/schools/gcsebitesize/maths

Remember: Do your best;
it is all you can do ☺